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MATHEMATICAL MODEL FOR ASSESSING THE RELIABILITY OF THE FUNCTIONING OF A DISTRIBUTION GAS SUPPLY NETWORK AS A QUEUING SYSTEM

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Abstract. *The paper posed and solved the problem of forming and formalizing a mathematical model for assessing the reliability of the operation of a gas distribution network as a queuing system. Difficulties associated with the formation of a mathematical model of the problem are due to taking into account a number of limitations a priori that arise in the format of uncontrolled random perturbations. The developed program for automating the calculation based on real production data "Hududgazta'minot Samarkand" allows you to determine the options for delivering the target product to consumers for minimal loss of time.*

Keywords: *model, mathematical model, reliability, estimation, functioning, network, gas supply, system, service.*

I. INTRODUCTION

Today, the methods of applied probability theory, stochastic processes, mathematical statistics, stochastic analysis, and queuing theory contribute to the resolution of the most complex problems, among which we can note the modern communication networks. The solution to these problems is to improve the availability and reliability of data transmission systems such as wireless communication networks. Today's wireless networks will reach a remarkable level of performance and efficiency in the years to come thanks to the progress of new mathematical models. One of the principal directions of creating the ultra-high-speed and reliable wireless communications within the framework of the development of future generation networks is the development of hybrid redundant systems based on laser and

radio technologies, each of which is sensitive to certain environmental conditions. A feature of such redundancy is that harsh weather conditions for one of the data transmissions channels do not affect the performance of the other channel. One of the main conditions for stable operation of the data transmission system is the insensitivity of quality and effectiveness of the system to the changes in the initial parameters of the model. The analysis of such sensitivity, which was carried out within the framework of this study, is one of the new areas of research of highly reliable next-generation data transmission systems, and it extends the previous studies in this area [1].

Previously [2], we performed the reliability assessment for a redundant system with arbitrary input distributions using the simulation model. In addition, we obtained the values of the relative

repair rate at which the desired level of reliability is achieved, and presented plots of the system uptime probability and plots of the empirical distribution function and the empirical reliability function.

In [3], the simulation approach was used to investigate the reliability of a homogeneous warm-standby data transmission system. Relative recovery rate (RRR) values at which the desired level of reliability is achieved were obtained with a relatively small excess of the mean availability values over the value of the repair time. A graphical analysis of the results obtained showed high asymptotic insensitivity of the empirical CDF and the system reliability function to the input distributions. In [4,5], it has been shown, respectively, that it is not always possible to obtain explicit analytical expressions for the SSP distribution of the homogeneous restorable data transmission system with hot and cold backup. The simulation model elaborated in this work made it possible to study the reliability at the system level, which is defined as the SSP of failure-free system operations, and also to assess the system reliability characteristics.

In [6], authors used the theory of uncertainty to study the mean time to failure (MTTF) and the reliability functions of the general system, within the framework of the reliability analysis of general systems with bi-uncertain variables. Using independent and non-identical uncertainty distributions with uncertain parameters for the lifetimes of system components, they presented the explicit expressions of the MTTF and the reliability function of each model. They also established and analyzed basic models of general systems with bi-uncertain variables. Authors of [7] carried out a mathematical model, using partial differential equations to investigate the

reliability of a two-component parallel repairable system with vacation. They assumed that component life follows exponential distribution and that component repair time and repairer's vacation time have general distribution, while the repairer goes on a vacation at the start. In [8], authors focused on the reliability analysis of a cold standby system under stepwise Poisson shocks, which come with intensity depending on the number of failures of the working element. They obtained an important reliability index for the system, using the supplementary variable method and the vector Markov process theory. Either internal factors or external shocks led to the system failure. Repair after breakage was assumed imperfect. In [9,10], authors investigated a reliability model of repairable systems with stochastic lifetimes and uncertain repair times. They established, respectively, the mathematical models of the reliability of repairable series systems/parallel/series-parallel/parallel-series systems. Li, J.; Chen, Y.; Yong, Z.; and Hung, H., [11] studied an availability model for a periodic inspection system with different lifetimes and repair-time distributions. They proposed an analytical and probabilistic model of availability for the considered system, using a new recursive algorithm that can provide limited average availability and instantaneous availability of a periodical inspection system with an arbitrary distribution of service life and repair time. Their approach can provide the technology support for improving system availability and determining a reasonable inspection period. Liu B.-y.; Feng Q.; Liu S.-q.; and Cai S.-w. [12], examined a reliability model with applications for calibration data. They developed a Wiener process model for modeling the calibration data by obtaining the unknown parameters of the model

using the maximum likelihood estimation approach. Lutfiah I. and Al T. [13] investigated the characteristics and application of the non-homogeneous Poisson process (NHPP) log-logistic reliability model. An NHPP reliability model based on the two-parameter log-logistic (LL) distribution was considered. Wang C. [14] investigated an approximate reliability analysis of large heterogeneous cold-standby systems. Abbas B.; Javad B.; and Tore M. [15] demonstrated the application of the available reliability models with covariates in the field of spare part predictions by means of a case study.

Furthermore, in other articles, it was reported that the use of one server and two identical units of the cold standby system with an imperfect switch is considered a semi-Markov process with three states. Such as in [16], Khorshidian, K. and Fathizadeh, M. explored an alternative approach to the reliability analysis of cold standby systems. Parshutina, S. and Bogatyrev, V. [17], demonstrated possibilities of the developed simulation models and means to support the computer-aided design of highly reliable distributed computer systems. The, J.; Lai, C.-M.; and Cheng, Y.-H. [18] presented a transmission line failure model and examined the effects of its parameters and the impact of the weather data correlation on the reliability performance of the considered system. Lisnianski, A.; Laredo, D.; and Haim, H.B. [19] performed a reliability analysis of a combined cycle gas turbine power plant by the means of a multi-state Markov model. Sensitivity analysis for reliability data uncertainty was also provided. Additionally, in [6-8], other authors have respectively focused their work on a “reliability analysis of cold standby compound system”, “reliability analysis

of a cold standby system attacked by shocks” and a “reliability analysis of a renewable multiple cold standby system”.

In the present article, we continue to perform the reliability study of a closed heterogeneous cold standby data transmission system with only one repair unit, with two different unreliable data sources, with an exponential CDF of uptime and a general independent CDF of the repair time of its components using the analytical approach [23]. System components (data sources) are subject to failure with different rates. Failures of system components are of a binary nature and can lead to different states of the system. Same as in the series of our previous works, in the current study we use the so-called Markovization method, which consists of introducing a supplementary variable that allows description of the behavior of the considered system by a two-dimensional Markov process. This approach allows the setting and proposing of the way to solve the problem of sensitivity analysis [24] of the system's output characteristics to the type of input distributions.

The process of formation and formalization of a mathematical model for assessing the reliability of the operation of a gas distribution network as a queuing system is due to a number of restrictions that require consideration, a priori, of uncontrolled random disturbances (accidents in sections, gas leakage, etc.). At the same time, deterministic variants of restrictions that determine the operating state of the network operation can change at any time. This makes it difficult to make decisions on the effective management of the network [11-17].

II. FORMULATION OF THE PROBLEM

The distribution gas supply network is investigated, consisting of M_i "requirements" - sections and gas supply facilities, where $i=\overline{1, n}$ and K_j - gas control devices (GRP), $j=\overline{1, l}$. The service time t_{ij} of each of the M_i sections, by each K_j device is set in the network operation mode.

If distribution gas supply networks are considered as a multi-stage system of the process of servicing the "requirements" of i - sections of the gas supply facility, including the last stages $r_i \geq 1$, then each section $i \in n$ and each stage $i \leq g \leq r_i$ of its service is assigned a certain number of devices $K_g^{(j)} \leq s$, $s = 1, 2, \dots, l$.

In the problem under consideration, the functioning of the distribution gas supply network is characterized by a flow type, where the time t_{ij} for servicing objects with K_j devices is set in accordance with the production need [12-15].

During the operation of the service system under consideration, an "interruption" with subsequent additional service is provided. At the same time, both arbitrary interruptions (the factor of uncontrolled random disturbances) and interruptions set in time - by production necessity are taken into account without fail.

If the service of the i - "requirement", j - "device" starts at time t_i^0 , then it proceeds continuously and ends at time

$$\bar{t} = t_i^0 + t_{il}.$$

Under the influence of a random factor, the process is interrupted and ends at the moment of time

$$\bar{t} = t_i^0 + t_{il} + \xi.$$

The process of servicing "requirements" in a deterministic system, i.e. in the normal operating mode of the network with the established technical and technological indicators, it is described by the schedule of its operation. At the same time, a certain set of indications is set in advance, which makes it possible to unambiguously determine which "requirements" by which particular "devices" were, are being served or will be served at each moment of time [4-8].

Under conditions in which the process of supplying gas to each device is set to a "hard" limitation, one should proceed from the assumption that each "requirement" cannot be simultaneously served by two or more devices and each device cannot simultaneously serve more than one "requirement".

Under this assumption, the schedule can be considered as a set of $R = \{R_1(t), R_2(t), \dots, R_n(t)\}$ piece-wise-constant left-continuous functions $R_l(t)$, $l = 1, n$, each of which is given on the time interval of the network operation and takes the values.

The most common and practically convenient way to estimate the quality of schedules for deterministic queuing systems is that each schedule R corresponds to a vector $\bar{t}(R) = (\bar{t}_1(R), \bar{t}_2(R), \dots, \bar{t}_n(R))$ of the moments of completion of servicing "requirements" for this schedule. Here, a real non-decreasing function of n variables $F(T) = F(t_1, \dots, t_n)$ is given, in which the quality of the schedule S is characterized by the value of this function at $T = \bar{t}(R)$. Of the two schedules, the one that corresponds to the lower $F(T)$ value is considered the best. The schedule that corresponds to the smallest $F(T)$ value is taken as the **optimal** one.

When the function $F(T)$ is given, each “requirement” i is usually associated with some function φ_i , that quantifies the “penalty” that must be “paid” if the service of this requirement is not completed at time t . The quality of the schedule S is characterized by the total or maximum penalty that must be paid when servicing the requirements on this schedule.

$$F_{\Sigma}(R) = \sum_{i=1}^n \varphi_i(\bar{t}_i(R, \xi))$$

or

$$F_{\max}(R) = \max_{1 \leq i \leq n} \{\varphi_i(\bar{t}_i(R, \xi))\}.$$

In particular, if $\varphi_i(\bar{t}) = t$, $i = \overline{1, n}$, then

$$F_{\max}(R) = \max_{1 \leq i \leq n} \{t(R, \xi)\}$$

is the moment of completion of servicing all requests (total service time). In this case, according to [*] $F_{\max}(R)$ is denoted by $\overline{t_{\max}}(R)$ and the schedule R^* that delivers the smallest value of $\overline{t_{\max}}(R)$ will be called the time-optimal schedule [16-18].

If $\varphi_i(t) = t - D_i$, then $F_{\max}(R)$ is denoted by $L_{\max}(R)$. Then we have

$$L_{\max}(R) = \max_{1 \leq i \leq n} \{L_i(R)\},$$

where $L_i(R) = \bar{t}_i(R) - D_i$ – is the offset time of the moment of completion of servicing the “requirement” relative to the due date D_i .

III. TASK ALGORITHM

According to the generated model of the queuing problem, the movement of the gas flow along the branches of the network in the established routes is considered. The gas flow is controlled by

K_j gas control devices (GCD), $j = \overline{1, l}$, during the time period t_{ij} . At the same time, the service time for gas supply facilities with K_j devices is set in accordance with the production need.

We also have time limits within the limits equal to $\frac{P_K}{P_N} < P_N$, $P_K \approx P_N$, respectively, the final and initial gas pressures.

It is assumed that the start of operation of the “device” for regulating the gas pressure in the network may be limited to the time of the working day or the time set by the production need. In addition, a certain “break” of connections in the service of “requirements” is also envisaged due to the manifestation of uncontrolled random disturbances [2,18].

Moreover, a certain “break” of connections in the “straight line” along the abscissa line in the XY coordinate system is also provided. This is due, for example, to a forced interruption of the gas supply (in the event of an accident at any section of the network), which is characterized by the time from 12:00 to 13:00.

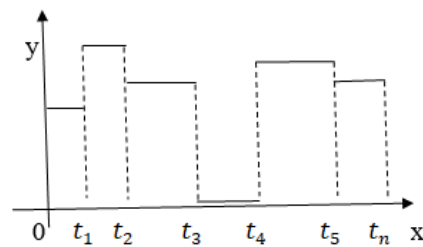


Fig.1. Controlling the gas flow of the queuing problem model.

- $t_3 - t_4$ – stop gas supply;
- t_4 – start time for gas supply countdown after a forced break;
- t_n – time of gas inflow to consumers;
- t_5 – time spent on gas inflow along the previously emergency route.

IV. BASICS OF USING THE PROGRAM

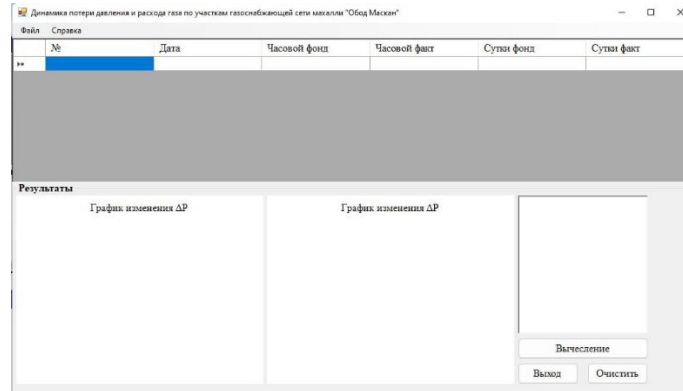


Fig. 2. The main interface of the program for automating the accounting of stock and hourly gas expenses.

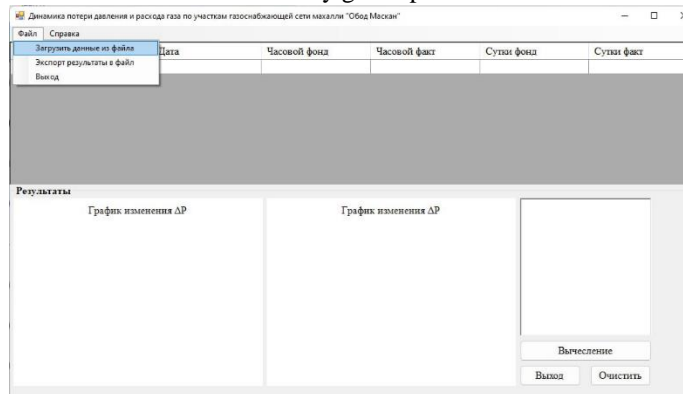


Fig. 3. The program interface of the process of loading data from files.

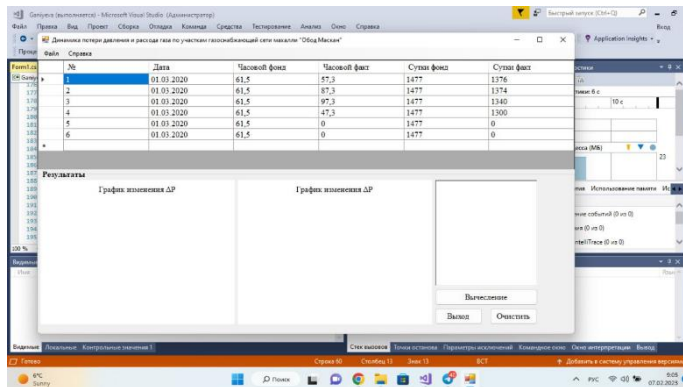


Fig. 4. The interface of the program for determining the undersupply of gas in the network sections by day for the period 03.13.2020-03.17.2020

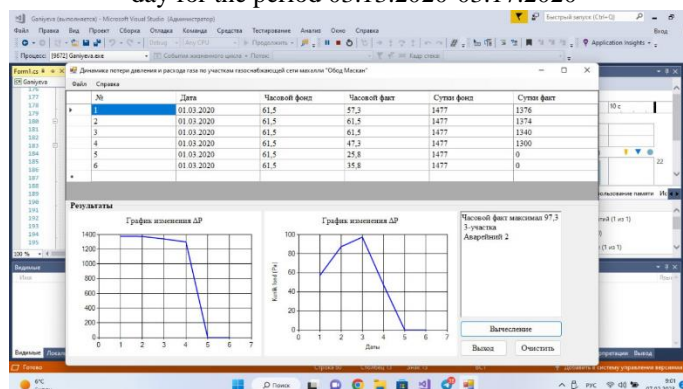


Fig. 5. Interface of the program for calculating the estimate of gas consumption.

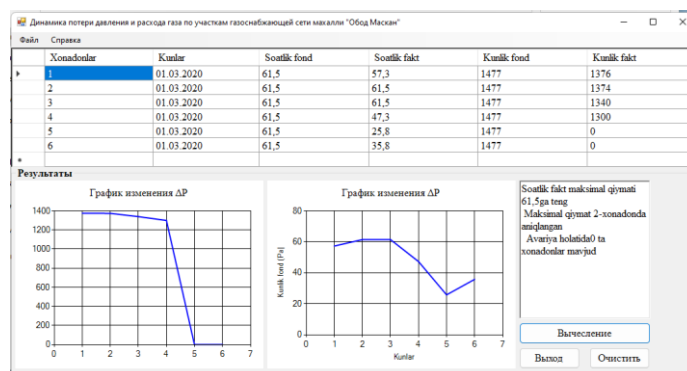


Fig. 6. Program interface for recalculated gas consumption estimate.

IV. CONCLUSION

A computational experiment for automating the calculation was carried out on the basis of real production data from the State Distribution Station "Samarkand1" and the State Distribution Station "Severny" of the organization "Hududgazta'minot Samarkand" for March 2020. According to the calculations, it was established that gas was not supplied on March 13.2020 - March 17.2020 in the corresponding sections of the gas supply network. Based on the analysis of the data of the hourly and daily gas funds, as well as their costs, it was determined by the software search to supply additional gas to these sections by the specified dates (03/13/2020 - 03/17/2020). The calculation automation program allows you to determine the options for delivering the target product to consumers for minimal loss of time.

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ОЦЕНКИ НАДЕЖНОСТИ ФУНКЦИОНИРОВАНИЯ РАСПРЕДЕЛИТЕЛЬНОЙ СЕТИ ГАЗОСНАБЖЕНИЯ КАК СИСТЕМЫ МАССОВОГО ОБСЛУЖИВАНИЯ

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Аннотация. В работе поставлена и решена задача формирования и формализации математической модели оценки надежности работы газораспределительной сети как системы массового обслуживания. Трудности, связанные с формированием математической модели задачи, обусловлены учетом ряда априорных ограничений, возникающих в формате неуправляемых случайных возмущений. Разработанная программа автоматизации расчета на основе реальных производственных данных «Худудгазтаъминот Самарканд» позволяет определить варианты доставки целевой продукции потребителям с минимальными потерями времени.

Ключевые слова: модель, математическая модель, надежность, оценка, функционирование, сеть, газоснабжение, система, обслуживание.

GAZ TAQSIMLASH TARMOG‘I ISHLASHINING ISHONCHLILIGINI BAHOLASHNING MATEMATIK MODEL

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Annotatsiya. Maqolada gaz taqsimlash tarmog‘i ishlashining ishonchliligini baholash uchun matematik modelini shakllantirish va rasmiylashtirish muammosi qo‘yilgan va hal qilingan. Muammoning matematik modelini shakllantirish bilan bog‘liq qiyinchiliklar nazoratsiz tasodifiy buzilishlar formatida paydo bo‘ladigan holatlari bir qator cheklovlarni hisobga olish bilan bog‘liq. Haqiqiy ishlab chiqarish ma‘lumotlari asosida hisob-kitoblarni avtomatlashtirish bo‘yicha ishlab chiqilgan “Hududgazta‘minot Samarqand” dasturi maqsadli mahsulotni iste‘molchilarga minimal vaqt yo‘qotgan holda yetkazib berish imkoniyatlarini aniqlash imkonini beradi.

Kalit so‘zlar: model, matematik model, ishonchlilik, baholash, ishlash, tarmoq, gaz ta‘minoti, tizim, xizmat ko‘rsatish.