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# IKKI QATLAMLI MUHITDA YER OSTI SUVLARI DINAMIKASI TA’SIRIDA TUZ KONSENTRATSIYASI O‘ZGARISH JARAYONINI MATEMATIK MODELLASHTIRISH

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**Annotatsiya**. Mazkur tadqiqotda ikki qatlamli geologik muhit sharoitida sizot va bosimli yer osti suvlari tarkibidagi tuz konsentratsiyasining o‘zgarish jarayonlari kompleks tarzda tahlil qilindi. Tadqiqot jarayonida atmosfera yog‘inlari, bug‘lanish jarayonlari, infiltratsiya oqimlari, suv olish intensivligi, qatlamlararo o‘tkazuvchanlik xususiyatlari, filtratsiya koeffitsiyenti, aktiv g‘ovaklik hamda qatlam qalinligi kabi muhim gidrogeologik omillarning tuz migratsiyasiga ta’siri hisobga olindi. Ushbu omillar ta’sirida yer osti suvlari muhitida konvektiv va diffuzion jarayonlari yuzaga kelib, tuzlarning yo‘nalishlar bo‘yicha taqsimlanishi va konsentratsiya gradientlari shakllanishi kuzatiladi. Tadqiqotda tuz konsentratsiyasining o‘zgarishi modda transporti jarayonlari asosida matematik jihatdan tavsiflandi. Modelda sizot va bosimli qatlamlar o‘rtasidagi massa almashinuvi hamda konsentratsiya gradientlari ta’sirida yuzaga keladigan modda ko‘chishi jarayonlari hisobga olindi. Bu yondashuv yer osti suvlari tarkibidagi tuzlarning harakatini real tabiiy jarayonlarga yaqin holda ifodalash imkonini beradi. Masala nochiziqli differensial tenglamalar tizimi orqali ifodalandi. Chegaraviy shartlarning murakkabligi sababli analitik yechim olish imkoniyati cheklanganligi bois, masalani yechish uchun yuqori aniqlikdagi sonli approksimatsiya usullaridan foydalanildi.

**Kalit so‘zlar:** yer osti suvlari gidrodinamikasi, geofiltratsiya, tuz konsentratsiyasi dinamikasi, sonli modellashtirish.

## 1 KIRISH

Yer osti suvlari insoniyat uchun strategik ahamiyatga ega tabiiy resurs bo‘lib, ichimlik suvi ta’minoti, irrigatsiya tizimlari hamda sanoat jarayonlarida asosiy manbalardan biri sifatida xizmat qiladi. Shu bilan birga, suv resurslarini baholashda faqat hajmiy ko‘rsatkichlar emas, balki ularning sifat parametrlari - ayniqsa minerallashuv darajasi va erigan tuzlar miqdori - barqaror rivojlanish konsepsiyasida muhim mezon hisoblanadi. Iqlim sharoitining o‘zgarishi, intensiv sug‘orish amaliyoti, meliorativ tadbirlar, shuningdek, urbanizatsiya va sanoat faoliyatining kengayishi yer osti suvlari sathining pasayishiga hamda sho‘rlanish jarayonlarining faollashuviga olib kelmoqda. Tuz konsentratsiyasining o‘zgarishi ko‘plab fizik, gidrodinamik va geokimyoviy omillar bilan shartlangan murakkab tizimli jarayondir. Suv sathining fazo-vaqt bo‘yicha o‘zgarishi, filtratsiya oqimining intensivligi, kapillyar ko‘tarilish, bug‘lanish jarayoni, qatlamlararo gidravlik bog‘liqlik hamda kimyoviy reaksiyalar minerallashuv dinamikasini shakllantiruvchi asosiy mexanizmlar sirasiga kiradi. Xususan, erkin (sizot) qatlamlarda suv sathi pasayganda kapillyar zona orqali yuqoriga ko‘tarilgan eritmalar bug‘lanish natijasida konsentratsiyalanadi va bu jarayon ikkilamchi sho‘rlanishni kuchaytiradi. Bosimli qatlamlarda esa tuz miqdorining o‘zgarishi ko‘proq zichlik gradientlari va qatlamlar o‘rtasidagi massa almashinuvi bilan belgilanadi.

Yer osti muhitida tuzlarning ko‘chishi asosan konvektiv va diffuzion transport mexanizmlari orqali amalga oshadi. Gidravlik gradient oqim yo‘nalishini va tezligini belgilasa, konsentratsiya gradienti diffuzion almashinuvni boshqaradi. Agar suyuqlik zichligi erigan modda miqdoriga sezilarli darajada bog‘liq bo‘lsa, tizimda zichlikka asoslangan konvektiv oqimlar yuzaga keladi, bu esa vertikal va gorizontal yo‘nalishlarda murakkab oqim tuzilmalarini shakllantiradi. Bunday sharoitlarda gidrodinamik va gidrokimyoviy jarayonlar bir-biridan ajralmas holda namoyon bo‘ladi. Tuz konsentratsiyasining yo‘nalishlar va vaqt bo‘yicha o‘zgarishini adekvat tavsiflash uchun faqat gidravlik bosim tenglamalariga tayanish yetarli emas. Massaning saqlanish tamoyiliga asoslangan transport tenglamalarini filtratsiya tenglamalari bilan birgalikda, o‘zaro bog‘langan tizim ko‘rinishida yechish talab etiladi. Ayniqsa, ko‘p qatlamli geologik tuzilmalarda suv sathi dinamikasi va minerallashuv jarayonlarini yagona matematik model doirasida integratsiyalash ilmiy nuqtai nazardan dolzarb masala hisoblanadi.

Yer osti suvlari va tuz migratsiyasi jarayonlarini modellashtirish zamonaviy gidrogeologiya fanining ustuvor ilmiy yo‘nalishlaridan biri hisoblanadi. Suv–tuz muvozanatini ifodalashda fizik asoslangan, zichlikka bog‘liq va integrallashgan matematik modellar keng qo‘llanilmoqda. Intensiv suv yig‘ilish havzalarida suv va tuz balansini fizik asoslangan integrallashgan model yordamida tavsiflash yondashuvi suv aylanish jarayonlarini yagona tizim sifatida ko‘rib chiqish imkonini berdi [1]. Zichlikka bog‘liq oqim va modda transportini matematik jihatdan asoslash hamda ularni verifikatsiya qilish bo‘yicha fundamental ishlar yer osti suvlari modellashtirish nazariyasining rivojlanishida muhim bosqich bo‘ldi [2]. Regional akviferlarda sho‘r suv intruziyasining kelib chiqishi va dinamikasini aniqlash maqsadida uch o‘lchamli sonli modellashtirish geofizik hamda geokimyoviy ma’lumotlar bilan integratsiya qilindi [3]. To‘yinmagan zonada suv va modda transportini fizik qonunlarga asoslangan neyron tarmoqlar yordamida modellashtirish esa klassik differensial tenglamalarga yangi yondashuv olib kirdi [4]. Qishloq xo‘jaligi hududlarida tuproq–suv–tuz o‘zaro ta’sirini mexanistik asosda ifodalash uchun modifikatsiyalangan SWAT modeli ishlab chiqildi [5]. Ifloslangan hududlarda gidrogeologik parametrlarni aniqlashda gidrotermal tuz-trasser sinovlariga asoslangan inversiya metodlari yuqori aniqlikni ta’minladi [6]. Sug‘orish drenaji jarayonida suv va tuz migratsiyasini sonli simulyatsiya qilish ekologik monitoring va resurslarni boshqarishda muhim ahamiyat kasb etadi [7]. Arid endoreik havzalarda zichlikka asoslangan yer osti suvlari dinamikasini sonli tahlil qilish murakkab gidrogeologik tizimlarni chuqurroq tushunish imkonini berdi [8]. Dengiz suvining qirg‘oqbo‘yi akviferlariga kirib borishini baholashda 3D sonli modellar samarali prognoz vositasi sifatida namoyon bo‘ldi [9]. Regional qishloq xo‘jaligi tizimlari uchun reaktiv transport va kimyoviy muvozanat modellarining ishlab chiqilishi minerallashuv jarayonlarini yanada realistik ifodalash imkonini berdi [10]. Konsentratsiyaga kuchli bog‘liq zichlikka ega suyuqliklar uchun oqim va modda ko‘chishini bog‘langan tizim sifatida ko‘rib chiqish konsepsiyasi esa yer osti suvlari sho‘rlanishini chuqurroq tushuntirishga xizmat qildi [11]. MODFLOW-LGR-MT3D platformasining yangi paketi regional miqyosda suv va tuz dinamikasini drenaj tizimlari bilan integratsiyalashgan holda modellashtirish imkonini kengaytirdi [12].

Sug‘oriladigan hududlarda sho‘r tuproqlarni rekultivatsiya qilish loyihalarining yer osti suvlari dinamikasiga ta’siri makon-vaqt kesimida baholandi [13]. Ko‘p qatlamli muhitlarda yer osti suvlari sathi o‘zgarishini matematik va sonli modellashtirish bo‘yicha tadqiqotlar qatlamlararo filtratsiya jarayonlarini chuqurroq tahlil qilish imkonini berdi [14]. Nolinear matematik model asosida suv sathi va tuz konsentratsiyasi o‘zgarishlarini prognozlash usullari ishlab chiqildi [15]. Yer osti suvlari va ularning tuz tarkibini kompleks o‘rganish natijasida bosimli va erkin qatlamlarda gidrokimyoviy jarayonlarning o‘ziga xos xususiyatlari aniqlangan [16]. Ikki qatlamli suv muhitida tuz konsentratsiyasi o‘zgarishini matematik modellashtirish esa konveksiya–diffuziya jarayonlarini qatlamlararo filtratsiya bilan integratsiyalash imkonini berdi [17]. Yer osti suvlari sathi va tuz konsentratsiyasi dinamikasini o‘rganishga bag‘ishlangan keyingi ishlarda matematik modellar takomillashtirilib, ularning amaliy qo‘llanish doirasi kengaytirildi [18,19]. Bir qatlamli muhit uchun ishlab chiqilgan uch o‘lchamli model esa ko‘p qatlamli tizimlar uchun nazariy asos vazifasini o‘tadi [20].

Amaliyot shuni ko‘rsatadiki, ko‘pincha suv resurslarini boshqarishda miqdoriy ko‘rsatkichlar ustuvor ahamiyat kasb etadi, suv sifati esa ikkilamchi parametr sifatida baholanadi. Biroq sug‘oriladigan hududlarda tuproq sho‘rlanishi, ichimlik suv manbalarining minerallashuvi hamda dengiz suvining akviferlarga kirib borishi kabi jarayonlar suv sifati omilini birlamchi mezon sifatida ko‘rib chiqishni taqozo etadi. Shu bois yer osti suvlari sathi va tuz konsentratsiyasi o‘zgarishlarini o‘zaro bog‘liq holda chuqur ilmiy asosda o‘rganish zamonaviy gidrogeologiyaning muhim yo‘nalishlaridan biri hisoblanadi.

## 2 MASALANI QO‘YILISHI

Mazkur tadqiqotda ikki qatlamli muhit uchun suv sathi va tuz konsentratsiyasi o‘zgarishini konveksiya-diffuziya va filtratsiya tenglamalari asosida bog‘langan tizim sifatida ifodalovchi matematik model taklif etiladi. Modelda zichlikka bog‘liq oqim, qatlamlararo massa almashinuvi hamda bug‘lanish jarayonlari hisobga olinadi. Bu yondashuv yer osti suvlari minerallashuvining makon-vaqt dinamikasini chuqurroq tushunish va amaliy prognozlash aniqligini oshirishga xizmat qiladi. Yer osti suvlari ya’ni sizot va bosimli suvlari sathi o‘zgarishini hisobga olgan holda yo‘nalishida ikki qatlamli g‘ovak muhitlarda tuz konsentratsiyasi o‘zgarish jarayoni quyidagi differensial tenglamalar sistemasi orqali ifodalaymiz:

$$\left.\begin{aligned}\rho_1 h\frac{\partial\theta_1}{\partial t}&=\frac{\partial}{\partial x}(D_1\rho_1 h\frac{\partial\theta_1}{\partial x})-\vartheta_1\rho_1 h\frac{\partial\theta_1}{\partial x},\\ \rho_2 H\frac{\partial\theta_2}{\partial t}&=\frac{\partial}{\partial x}(D_2\rho_2 H\frac{\partial\theta_2}{\partial x})-\vartheta_2\rho_2 H\frac{\partial\theta_2}{\partial x}.\end{aligned}\right\}\qquad(1)$$

Boshlang‘ich va chegaraviy shartlar:

$$\theta_1(x,0)=(\theta_1)_0,\quad \theta_2(x,0)=(\theta_2)_0, \tag{2}$$

$$\rho_1 h\frac{\partial\theta_1}{\partial x}\Big|_{x=0}=0,\ (\vartheta_1\theta_1-D_1\rho_1 h\frac{\partial\theta_1}{\partial x})\Big|_{x=L_x}=0, \tag{3}$$

$$\rho_2 H\frac{\partial\theta_2}{\partial x}\Big|_{x=0}=0,\ (\vartheta_2\theta_2-D_2\rho_2 H\frac{\partial\theta_2}{\partial x})\Big|_{x=L_x}=0, \tag{4}$$

$$\theta_1(x,t)\big|_{x=m_x-0}=\theta_2(x,t)\big|_{x=m_x+0}, \tag{5}$$

$$\vartheta_1\theta_1-D_1\rho_1 h\frac{\partial\theta_1}{\partial x}\bigg|_{x=m_x+0}=\vartheta_2\theta_2-D_2\rho_2 H\frac{\partial\theta_2}{\partial x}\bigg|_{x=m_x-0}. \tag{6}$$

bu yerda $\theta_1(x,t)$, $\theta_2(x,t)$ – sizot va bosimli suvlardagi tuz konsentratsiyalari; $\vartheta_1$, $\vartheta_2$ – filtratsiya tezliklari; $D_1,\ D_2$ – diffuziya koeffitsiyentlari; $(\theta_1)_0$, $(\theta_2)_0$ – tuz konsentratsiyalarining boshlang‘ich qiymatlari; $\rho_0$, $\rho_1$ - sizot suvidagi va bosimli suvdagi tuz konsentratsiyalari zichligi.

## 3 MASALANI YECHILISHI

Quyilgan masalani yechish uchun ushbu

$$(\theta_1)^*=\frac{\theta_1}{(\theta_1)_0},\ (\theta_2)^*=\frac{\theta_2}{(\theta_2)_0},\ x^*=\frac{x}{L},\ (D_1)^*=\frac{D_1}{(D_1)_0},\ (D_2)^*=\frac{D_2}{(D_2)_0},\ \tau=\frac{D_0}{L^2}t,\ (\rho_1)^*=\frac{\rho_1}{(\rho_1)_0},$$

$$(\rho_2)^*=\frac{\rho_2}{(\rho_2)_0},\ (\vartheta_1)^*=\frac{\vartheta_1}{(\vartheta_1)_0},\ (\vartheta_2)^*=\frac{\vartheta_2}{(\vartheta_2)_0}.$$

o‘lchovsiz kattaliklarni kiritamiz. Keyingi hisoblash jarayonlarida qulaylik bo‘lishi uchun «*» belgisini tushirib qoldirib (1) tenglama va (3) – (6) chegaraviy shartlarni quyidagi ko‘rinishda yozamiz:

$$\left.\begin{aligned}&\rho_1 h\frac{\partial\theta_1}{\partial\tau}=\frac{\partial}{\partial x}(D_1\rho_1 h\frac{\partial\theta_1}{\partial x})-\xi\vartheta_1\rho_1 h\frac{\partial\theta_1}{\partial x},\\&\rho_2 H\frac{\partial\theta_2}{\partial\tau}=\xi_1\frac{\partial}{\partial x}(D_2\rho_2 H\frac{\partial\theta_2}{\partial x})-\xi_2\vartheta_2\rho_2 H\frac{\partial\theta_2}{\partial x}.\end{aligned}\right\} \tag{7}$$

bu yerda $\xi=\dfrac{(\vartheta_1)_0 L}{(D_1)_0}$, $\xi_1=\dfrac{(D_2)_0}{(D_1)_0}$, $\xi_2=\dfrac{(\vartheta_2)_0 L}{(D_1)_0}$.

$$(\theta_1)_{t=0}=(\theta_1)_0,\quad (\theta_2)_{t=0}=(\theta_2)_0, \tag{8}$$

$$\frac{(\rho_1)_0(\theta_1)_0 h_0}{L}\rho_1 h\frac{\partial\theta_1}{\partial x}\Big|_{x=0}=0,\quad ((\vartheta_1)_0(\theta_1)_0\vartheta_1\theta_1-\frac{(\rho_1)_0(D_1)_0(\theta_1)_0 h_0}{L}D_1\rho_1 h\frac{\partial\theta_1}{\partial x})\Big|_{x=1}=0, \tag{9}$$

$$\frac{(\rho_2)_0(\theta_2)_0 H_0}{L}\rho_2 H\frac{\partial\theta_2}{\partial x}\Big|_{x=0}=0,\ ((\vartheta_2)_0(\theta_2)_0\vartheta_2\theta_2-\frac{(\rho_2)_0(D_2)_0(\theta_2)_0 H_0}{L}D_2\rho_2 H\frac{\partial\theta_2}{\partial x})\Big|_{x=1}=0, \tag{10}$$

$$\theta_1\Big|_{x=\frac{m_x-0}{L}}=\theta_2\Big|_{x=\frac{m_x+0}{L}}\ , \tag{11}$$

$$\begin{aligned}&((\vartheta_1)_0(\theta_1)_0\vartheta_1\theta_1-\frac{(\rho_1)_0(D_1)_0(\theta_1)_0 h_0}{L}D_1\rho_1 h\frac{\partial\theta_1}{\partial x})\bigg|_{x=\frac{m_x-0}{L}}=\\&=((\vartheta_2)_0(\theta_2)_0\vartheta_2\theta_2-\frac{(\rho_2)_0(D_2)_0(\theta_2)_0 H_0}{L}D_2\rho_2 H\frac{\partial\theta_2}{\partial x})\bigg|_{x=\frac{m_x+0}{L}},\end{aligned} \tag{12}$$

(7) – (12) masalalarni sonli yechish uchun chekli ayirmalar usulini qo‘llaymiz. Bu masalalarni sonli yechishda yuqori approksimatsiya aniqligiga ega bo‘lgan absolyut turg‘un oshkormas sxemadan foydalanamiz. Buning uchun $D=\{0\le x<L,\ 0\le t\le J\}$ sohada noma’lumning o‘zgarish maydonining chegaraviy shartlarini hisobga olgan holda $\Delta x,\ \Delta\tau$ qadamlarga mos to‘rni kiritamiz [21-28]:

$$\omega_{\Delta x,\Delta\tau}=\{(x_i,t_j),\ x_i=i\Delta x;\ i=0,1,2,\dots I;\ \ t_j=j\Delta\tau;\ j=0,1,2,\dots,J\}$$

Quyida (7) tenglamalar sistemasini $\omega_{\Delta x,\Delta\tau}$ to‘rdan foydalangan holda oshkormas chekli ayirmali sxema ko‘rinishida approksimatsiya qilamiz:

$$\left.\begin{aligned}
&\left(\frac{A}{\Delta x^2}(D_1)_{i-0.5}h_{i-0.5}^{j+1}+\frac{A_1h_i^{j+1}}{2\Delta x}(|v_x|+v_x)\right)(\theta_1)_{i-1}^{j+1}-\left(\frac{h_i^{j+1}}{\Delta\tau}+\frac{A}{\Delta x^2}((D_1)_{i-0.5}+(D_1)_{i+0.5})(h_{i-0.5}^{j+1}+h_{i+0.5}^{j+1})+\right.\\
&\left.+\frac{A_1h_i^{j+1}}{\Delta x}v_x\right)(\theta_1)_i^{j+1}+\left(\frac{A}{\Delta x^2}(D_1)_{i+0.5}h_{i+0.5}^{j+1}-\frac{A_1h_i^{j+1}}{2\Delta x}(|v_x|-v_x)\right)(\theta_1)_{i+1}^{j+1}=-\left(\frac{h_i^{j+1}}{\Delta\tau}(\theta_1)_i^{j}-A_2f_1\theta_{1f}\right),\\
&\left(\frac{B}{\Delta x^2}(D_2)_{i-0.5}H_{i-0.5}^{j+1}+\frac{B_1H_i^{j+1}}{2\Delta x}(|v_x|+v_x)\right)(\theta_2)_{i-1}^{j+1}-\left(\frac{H_i^{j+1}}{\Delta\tau}+\frac{B}{\Delta x^2}((D_2)_{i-0.5}+(D_2)_{i+0.5})(H_{i-0.5}^{j+1}+H_{i+0.5}^{j+1})+\right.\\
&\left.+\frac{B_1H_i^{j+1}}{\Delta x}v_x\right)(\theta_2)_i^{j+1}+\left(\frac{B}{\Delta x^2}(D_2)_{i+0.5}H_{i+0.5}^{j+1}-\frac{B_1H_i^{j+1}}{2\Delta x}(|v_x|-v_x)\right)(\theta_2)_{i+1}^{j+1}=-\left(\frac{H_i^{j+1}}{\Delta\tau}(\theta_2)_i^{j}+B_2f_2\theta_{2f}\right).
\end{aligned}\right\}\quad(13)$$

(13) chekli ayirmali sistemani uch diagonalli chiziqli algebraik tenglamalar sistemasi orqali quyidagicha ifodalaymiz:

$$\bar a_i(\theta_1)_{i-1}^{j+1}-\bar b_i(\theta_1)_i^{j+1}+\bar c_i(\theta_1)_{i+1}^{j+1}=-\bar d_i,\tag{14}$$

$$\bar a_i^1(\theta_2)_{i-1}^{j+1}-\bar b_i^1(\theta_2)_i^{j+1}+\bar c_i^1(\theta_2)_{i+1}^{j+1}=-\bar d_i^1.\tag{15}$$

(14) va (15) tenglamalar sistemasidagi koeffitsiyentlar va ozod hadlar quyidagi munosabatlar orqali topiladi:

$$\bar a_i=\frac{A}{\Delta x^2}(D_1)_{i-0.5}h_{i-0.5}^{j+1}+\frac{A_1h_i^{j+1}}{2\Delta x}(|v_x|+v_x),$$

$$\bar b_i=\frac{h_i^{j+1}}{\Delta\tau}+\frac{A}{\Delta x^2}((D_1)_{i-0.5}+(D_1)_{i+0.5})(h_{i-0.5}^{j+1}+h_{i+0.5}^{j+1})+\frac{A_1h_i^{j+1}}{\Delta x}v_x,$$

$$\bar c_i=\frac{A}{\Delta x^2}(D_1)_{i+0.5}h_{i+0.5}^{j+1}-\frac{A_1h_i^{j+1}}{2\Delta x}(|v_x|-v_x),$$

$$\bar d_i=\frac{h_i^{j+1}}{\Delta\tau}(\theta_1)_i^j-A_2f_1\theta_{1f},\ \ \bar a_i^1=\frac{B}{\Delta x^2}(D_2)_{i-0.5}H_{i-0.5}^{j+1}+\frac{B_1H_i^{j+1}}{2\Delta x}(|v_x|+v_x),$$

$$\bar b_i^1=\frac{H_i^{j+1}}{\Delta\tau}+\frac{B}{\Delta x^2}((D_2)_{i-0.5}+(D_2)_{i+0.5})\left(H_{i-0.5}^{j+1}+H_{i+0.5}^{j+1}\right)+\frac{B_1H_i^{j+1}}{\Delta x}v_x,$$

$$\bar c_i^1=\frac{B}{\Delta x^2}(D_2)_{i+0.5}H_{i+0.5}^{j+1}-\frac{B_1H_i^{j+1}}{2\Delta x}(|v_x|-v_x),\ \ \bar d_i^1=\frac{H_i^{j+1}}{\Delta\tau}(\theta_2)_i^j+B_2f_2\theta_{2f}.$$

(14) va (15) tenglamalar sistemasini haydash usuli yordamida yechamiz. Bu holda quyidagi rekurrent formulalardan foydalanamiz:

$$(\theta_1)_i^{j+1}=\bar\alpha_{i+1}(\theta_1)_{i+1}^{j+1}+\bar\beta_{i+1},\tag{16}$$

$$(\theta_2)_i^{j+1}=\bar\alpha_{i+1}^1(\theta_2)_{i+1}^{j+1}+\bar\beta_{i+1}^1,\tag{17}$$

bu yerda $\bar\alpha_i$, $\bar\beta_i$, $\bar\alpha_i^1$, $\bar\beta_i^1$ – haydash koeffitsiyentlari bo‘lib, quyidagi formulalar orqali topiladi:

$$\bar\alpha_{i+1}=\frac{\bar c_i}{\bar b_i-\bar a_i\bar\alpha_i},\ \ \bar\beta_{i+1}=\frac{\bar d_i+\bar a_i\bar\beta_i}{\bar b_i-\bar a_i\bar\alpha_i},\ \ \bar\alpha_{i+1}^1=\frac{\bar c_i^1}{\bar b_i^1-\bar a_i^1\bar\alpha_i^1},\ \ \bar\beta_{i+1}^1=\frac{\bar d_i^1+\bar a_i^1\bar\beta_i^1}{\bar b_i^1-\bar a_i^1\bar\alpha_i^1}.$$

(3) – (6) chegaraviy shartlarni approksimatsiya qilamiz:

$$\frac{\mu h_0(\theta_1)_0}{L}h_1^{j+1}\frac{(\theta_1)_0^{j+1}-4(\theta_1)_1^{j+1}+3(\theta_1)_2^{j+1}}{2\Delta x}=-((\theta_1)_0(\theta_1)_1^{j+1}-(\theta_1)_0),$$

$$\frac{\mu h_0(\theta_1)_0}{L}h_I^{j+1}\frac{-3(\theta_1)_{I-1}^{j+1}+4(\theta_1)_I^{j+1}-(\theta_1)_{I+1}^{j+1}}{2\Delta x}=((\theta_1)_0(\theta_1)_I^{j+1}-(\theta_1)_0),$$

$$\frac{\mu^* H_0(\theta_2)_0}{L}H_1^{j+1}\frac{(\theta_2)_0^{j+1}-4(\theta_2)_1^{j+1}+3(\theta_2)_2^{j+1}}{2\Delta x}=-((\theta_2)_0(\theta_2)_1^{j+1}-(\theta_2)_0),$$

$$\frac{\mu^* H_0(\theta_2)_0}{L}H_I^{j+1}\frac{-3(\theta_2)_{I-1}^{j+1}+4(\theta_2)_I^{j+1}-(\theta_2)_{I+1}^{j+1}}{2\Delta x}=((\theta_2)_0(\theta_2)_I^{j+1}-(\theta_2)_0),$$

$$(\theta_1)_0(\theta_1)_{\frac{I}{2}}^{j}=(\theta_2)_0(\theta_2)_{\frac{I}{2}}^{j},$$

$$\frac{(D_1)_0 h_0(\theta_1)_0}{L}(D_1)_{\frac{I}{2}}h_{\frac{I}{2}}^{j+1}\frac{-3(\theta_1)_{I-1}^{j+1}+4(\theta_1)_I^{j+1}-(\theta_1)_{I+1}^{j+1}}{2\Delta x}=$$

$$=\frac{(D_2)_0 H_0(\theta_2)_0}{L}(D_2)_{\frac{I}{2}}H_{\frac{I}{2}}^{j+1}\frac{-3(\theta_2)_{I-1}^{j+1}+4(\theta_2)_I^{j+1}-(\theta_2)_{I+1}^{j+1}}{2\Delta x}.$$

Uch dioganalli tenglamalar sistemasi, rekurent ifodalar hamda chegaraviy shartlardanfoydalanib $\bar{\alpha}_1$, $\bar{\beta}_1$, $\bar{\alpha}_1^1$, $\bar{\beta}_1^1$ – haydash koeffitsiyentlarining dastlabki qiymatlarini hamda tuz konsentratsiyasining chegaradagi qiymatini topamiz:

$$\bar{\alpha}_1=\frac{3\mu h_0 h_1^{j+1}\bar{b}_1-4\mu h_0 h_1^{j+1}\bar{c}_1+2\Delta x L\bar{c}_1}{\mu h_0 h_1^{j+1}(3\bar{a}_1-\bar{c}_1)},\quad \bar{\beta}_1=-\frac{2\Delta x L\bar{c}_1+3\mu h_0 h_1^{j+1}\bar{d}_1}{\mu h_0 h_1^{j+1}(3\bar{a}_1-\bar{c}_1)}.$$

$$(\theta_1)_I^{j+1}=\frac{\bar{\beta}_I\mu h_0 h_I^{j+1}(3\bar{c}_I-\bar{a}_I)-\bar{d}_I\mu h_0 h_I^{j+1}+2\Delta x L\bar{c}_I}{4\mu h_0 h_I^{j+1}\bar{c}_I-2\Delta x L\bar{c}_I-\mu h_0 h_I^{j+1}\bar{b}_I-\bar{\alpha}_I\mu h_0 h_I^{j+1}(3\bar{c}_I-\bar{a}_I)}.$$

$$\bar{\alpha}_1^1=\frac{4\mu^* H_0 H_1^{j+1}\bar{c}_1^1-2\Delta x L\bar{c}_1^1-\bar{b}_1^1 3\mu^* H_0 H_1^{j+1}}{\mu^* H_0 H_1^{j+1}(\bar{c}_1^1-3\bar{a}_1^1)},\quad \bar{\beta}_1^1=\frac{2\Delta x L\bar{c}_1^1+\bar{d}_1^1 3\mu^* H_0 H_1^{j+1}}{\mu^* H_0 H_1^{j+1}(\bar{c}_1^1-3\bar{a}_1^1)},$$

$$(\theta_2)_I^{j+1}=\frac{\bar{\beta}_I^1\mu^* H_0 H_I^{j+1}(\bar{a}_I^1-3\bar{c}_I^1)+2\Delta x L\bar{c}_I^1+\mu^* H_0 H_I^{j+1}\bar{d}_I^1}{\mu^* H_0 H_I^{j+1}\bar{b}_I^1-4\mu^* H_0 H_I^{j+1}\bar{c}_I^1+2\Delta x L\bar{c}_I^1-\bar{\alpha}_I^1\mu^* H_0 H_I^{j+1}(\bar{a}_I^1-3\bar{c}_I^1)}.$$

Teskari haydash usulidan foydalanib $(\theta_1)_{I-1}^j, (\theta_1)_{I-2}^j,..., (\theta_1)_1^j$, $(\theta_2)_{I-1}^j, (\theta_2)_{I-2}^j,..., (\theta_2)_1^j$ sizot va bosimli suvli qatlamdagi tuz konsentratsiyasining hamda tuz konsentratsiyasining qiymatlarini topamiz.

## 4 XULOSA

Tadqiqot jarayonida yer osti suvlari tarkibidagi tuz konsentratsiyasining turli yo‘nalishlar va vaqt davomida o‘zgarish qonuniyatlarini aniqlash maqsadida matematik hamda sonli modellashtirish yondashuvlaridan foydalanildi. Olingan natijalar shuni ko‘rsatdiki, bunday modellashtirish usullari ko‘p qatlamli gidrogeologik tizimlarda modda migratsiyasi jarayonlarini ishonchli baholash va ularning rivojlanish tendensiyalarini aniqlash imkonini beradi. Tadqiqot doirasida ikki qatlamli geologik muhitda sizot hamda bosimli yer osti suvlari tarkibida tuzlarning tarqalish mexanizmlari tizimli ravishda o‘rganildi va ushbu jarayonlarga ta’sir etuvchi asosiy gidrogeologik omillar aniqlab berildi. Xususan, filtratsiya jarayonlari, qatlamlararo modda almashinuvi, infiltratsiya oqimlari, bug‘lanish jarayonlari, g‘ovak muhitning fizik xossalari hamda gidravlik gradientlar tuz konsentratsiyasining o‘zgarish dinamikasini shakllantiruvchi asosiy omillar sifatida ko‘rib chiqildi.

Tahlillar shuni ko‘rsatdiki, yer osti suvlari tizimida tuzlarning ko‘chishi asosan konvektiv va diffuzion jarayonlari orqali amalga oshadi. Qatlamlararo gidrodinamik bog‘lanishlar natijasida esa konsentratsiya maydonining turli yo‘nalishlarda qayta taqsimlanishi yuz beradi. Ushbu jarayonlarni nazariy jihatdan tavsiflash uchun konveksiya–diffuziya tipidagi tenglamalarga asoslangan matematik model shakllantirildi. Model tuz konsentratsiyasining o‘zgarishini gidrodinamik omillar bilan integrallashgan holda ifodalash imkonini berib, ikki qatlamli muhitda modda transportining asosiy mexanizmlarini tavsiflashga xizmat qiladi. Murakkab chegaraviy shartlar mavjudligi sababli analitik yechimni aniqlash amaliy jihatdan qiyin bo‘lganligi bois, masalani yechishda yuqori aniqlikka ega bo‘lgan sonli hisoblash usullari qo‘llanildi.

Hisoblash jarayonida iteratsion algoritmlar hamda barqaror hisoblash sxemalaridan foydalanildi. Natijalar shuni koʻrsatadiki, yer osti suvlari tarkibidagi tuz konsentratsiyasining oʻzgarishi gidrodinamik jarayonlar bilan uzviy bogʻliq boʻlib, ularning yoʻnalishlar va vaqt boʻyicha rivojlanishi koʻplab tabiiy omillarning oʻzaro taʼsiri natijasida shakllanadi. Taklif etilgan matematik model ushbu jarayonlarni kompleks tahlil qilish, minerallashuv darajasining kelgusidagi oʻzgarishlarini prognozlash hamda suv resurslarining sifat holatini baholash uchun samarali ilmiy-uslubiy asos boʻlib xizmat qiladi. Bundan tashqari, mazkur yondashuv sugʻoriladigan hududlarda shoʻrlanish jarayonlarini nazorat qilish, ekologik monitoringni tashkil etish hamda suv resurslaridan oqilona foydalanish strategiyalarini ishlab chiqishda muhim amaliy ahamiyatga ega.

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# MATHEMATICAL MODELLING OF SALT CONCENTRATION VARIATIONS UNDER THE INFLUENCE OF GROUNDWATER DYNAMICS IN A TWO-LAYER MEDIUM

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**Abstract**. This study presents a comprehensive analysis of the variation of salt concentration in groundwater within a two-layer geological medium, taking into account both unconfined and confined aquifers. The investigation considers the influence of several significant hydrogeological factors affecting salt migration, including atmospheric precipitation, evaporation processes, infiltration flows, groundwater extraction intensity, interlayer permeability characteristics, filtration coefficients, effective porosity, and aquifer thickness. Under the influence of these factors, convective and diffusive transport mechanisms arise within the groundwater system, leading to the spatial redistribution of dissolved salts and the formation of concentration gradients. In the present research, the dynamics of salt concentration are described mathematically based on the fundamental principles of mass transport in porous media. The proposed model incorporates the processes of mass exchange between the unconfined and confined aquifer layers, as well as solute transport driven by concentration gradients. Such an approach enables a realistic representation of salt migration processes occurring in groundwater systems under natural hydrogeological conditions. The governing processes are formulated in the form of a system of nonlinear differential equations. Due to the complexity of the boundary conditions, obtaining an analytical solution is difficult; therefore, high-accuracy numerical approximation methods are employed to obtain reliable computational solutions for the problem.

**Keywords:** groundwater hydrodynamics, geofiltration processes, salt concentration dynamics, numerical modelling.

# МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА ИЗМЕНЕНИЯ КОНЦЕНТРАЦИИ СОЛЕЙ ПОД ВЛИЯНИЕМ ДИНАМИКИ ПОДЗЕМНЫХ ВОД В ДВУХСЛОЙНОЙ СРЕДЕ

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**Аннотация**. В данном исследовании проведён комплексный анализ процессов изменения концентрации солей в составе грунтовых и напорных подземных вод в условиях двухслойной геологической среды. В ходе исследования учтено влияние важных гидрогеологических факторов, таких как атмосферные осадки, процессы испарения, инфильтрационные потоки, интенсивность водоотбора, межслойная проницаемость, коэффициент фильтрации, активная пористость и мощность водоносного слоя, на миграцию солей. Под воздействием указанных факторов в среде подземных вод возникают конвективные и диффузионные процессы, в результате чего наблюдается перераспределение солей по направлениям и формирование градиентов концентрации. В работе изменение концентрации солей математически описано на основе процессов переноса вещества. В предложенной модели учтены процессы массопереноса между безнапорным и напорным водоносными горизонтами, а также перенос вещества, обусловленный градиентами концентрации. Такой подход позволяет описывать движение растворённых солей в подземных водах с учётом реальных природных гидрогеологических условий. Задача сформулирована в виде системы нелинейных дифференциальных уравнений. В связи со сложностью граничных условий получение аналитического решения ограничено, поэтому для решения задачи применены высокоточные численные методы аппроксимации.

**Ключевые слова:** гидродинамика подземных вод, геофильтрация, динамика концентрации солей, численное моделирование.