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DINAMIK BOG‘LANGAN IKKI O‘LCHOVLI TERMO-ELASTIKLIK VA TERMO-PLASTIKLIK NAZARIYASI MASALALARINI SONLI YECHISH

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Annotatsiya. Maqolada Ilyushinning deformatsion plastiklik nazariyasiga asoslangan holda ikki o‘lchovli bog‘langan dinamik termo-plastik chegaraviy masalani yechishning sonli usuli ko‘rib chiqilgan. Diskret tenglamalar chekli-ayirmalar usuli yordamida oshkor va oshkormas sxemalar ko‘rinishida tuzilgan. Oshkor sxemalarning yechimi ko‘chish va harorat komponentlariga nisbatan rekurrent munosabatlarga keltirilgan. Oshkormas sxemalar mos yo‘nalishlar bo‘yicha uch diagonal matritsali sistemalar uchun haydash usulidan foydalanib samarali yechilgan. Bunda matritsalarining diagonal ustunligi oshkormas chekli-ayirmali sxemalarning yaqinlashishini ta’minlagan. Ichki issiqlik maydoni ta’sirida har tomondan siqilgan termo-plastik to‘g‘ri to‘rtburchak masalasi sonli usulda yechilgan. Termo-plastik to‘g‘ri to‘rtburchakning kuchlanganlik-deformatsiyalanganlik holati va vaqt bo‘yicha turli kesimlar va nuqtalar bo‘yicha ko‘chish va haroratning taqsimlanishi o‘rganilgan.

Kalit so‘zlar: termo-plastiklik, ko‘chis, temperatura, kuchlanish, chekli-ayirmali tenglamalar, oshkor sxema, yaqinlashish.

1 KIRISH

Ilm-fan va texnika taraqqiyotining hozirgi bosqichida konstruksiyalar va ularning elementlarining kuchlanganlik-deformatsiyalanganlik holatini o‘rganish, ularning mustahkamlik va ishonchlilik chegaralarini aniqlash, termomexanik elastik-plastik deformatsiyalarni hisobga olish ilmiy-texnik tatbiqlarning dolzarb vazifasidir. Issiqlikning tarqalish jarayonini tavsiflovchi matematik modellar birinchi marta Duhamel-Neumann [3] ishlarida ko‘rib chiqilgan bo‘lib, unda to‘liq deformatsiya elastik deformatsiya va issiqlik kengayishidan iborat deb taxmin qilingan.

Termo-plastiklik nazariyasi masalalari birinchi marta P.M. Nagdining ishlarida batafsil ko‘rib chiqilgan va umumiy deformatsiya elastik, plastik va issiqlik deformatsiyalaridan iborat deb taxmin qilingan. Keyinchalik bu tadqiqotlar Yu.N. Shevchenko, D. Kolarov, B. Pobedri, J. Cesey, J. Aboudi, V.F. Chen va boshqalarning ishlarida davom ettirildi. Termo-plastiklikning bog‘langan masalalari texnik qo‘llanmalar uchun juda muhim. Termodinamika qonunlari doirasida bog‘langan chegaraviy masalalar birinchi marta M. Biot, V. Novatskiy, B. Pobedri, Youssef va boshqalarning ishlarida ko‘rib chiqilgan. Chekli deformatsiyalar bilan bog‘liq muammolar J.C. Simo va C. Miehe, Z. Sloderbach va J. Pajak, M. Vaz va B.A. Munoz-Rojas va boshqalarning ishlarida ko‘rib chiqilgan. Termo-elastiklik va termo-plastiklik masalalarini yechishda, odatda, issiqlik oqimi tenglamasini yechish asosida harorat taqsimotlari, termo-elastiklik va termo-plastiklik chegaraviy masalalarini yechishda esa hajmiy kuchlar bilan birgalikda qaraladigan harorat hadlari oldindan aniqlangan.

So‘nggi yillarda termo-elastik-plastik deformatsiyalarning paydo bo‘lishiga issiqlik va mexanik omillarning o‘zaro ta’sirini o‘rganishga bag‘ishlangan ilmiy tadqiqotlar jadal sur’atlar bilan o‘sib bormoqda. Termo-mexanik kuchlarning o‘zaro ta’sirini hisobga olish issiqlik oqimi tenglamasini bog‘langan termo-elastik-plastik qattiq jismlarning termodinamik tenglamalari bilan birgalikda ko‘rib chiqish orqali amalga oshirilishi mumkin. Odatda, bu masalalar elastiklik va termo-plastiklik nazariyasining bog‘langan masalalari deb ataladi [6-8, 10]. Bu yerda termo-elastik-plastiklik termini termo-elastiklik va termo-plastiklik nazariyasining chegaraviy masalalarini anglatadi. Bog‘langan termo-elastik-plastik masalalarni yechishning asosiy sonli usullari chekli elementlar usuli, chekli-differensial usullar va boshqalar hisoblanadi. Keyingi vaqtlarda chegaraviy elementlar usuli keng qo‘llanilmoqda.

Ushbu ishda izotrop jismlarning Ilyushinning deformatsion nazariyasiga asoslangan termo-plastiklikning ikki o‘lchovli bog‘langan dinamik masalasi sonli yechilgan. Diskret tenglamalar chekli-ayirmalar usuli bilan oshkor va oshkormas sxemalar ko‘rinishida tuziladi. Oshkor sxemalarning yechimi ko‘chish va harorat komponentlariga nisbatan rekurrent munosabatlarga keltiriladi. Oshkormas sxemalar tegishli yo‘nalishlar bo‘yicha haydash usulini qo‘llashga keltirilgan.

2 MASALANING QO‘YILISHI

Harakat tenglamasidan iborat bo‘lgan bog‘langan termo-elastik-plastik masalaning matematik modelini ko‘rib chiqamiz [4]:

$$\sum_{j=1}^{3}\frac{\partial\sigma_{ij}}{\partial x_j}+X_i=\rho\frac{\partial^2 u_i}{\partial t^2},\ \ i=1,3. \tag{1}$$

Ilyushinning deformatsion nazariyasining konstitutiv tenglamasi [1]:

$$\sigma_{ij}=\sigma\delta_{ij}+\frac{\sigma_u}{\varepsilon_u}e_{ij}, \tag{2}$$

$$\sigma=K(\theta-3\alpha\vartheta), \tag{3}$$

$$\sigma_u=\sigma_u(\varepsilon_u,T). \tag{4}$$

Izotrop jismlar uchun issiqlik oqimi tenglamasi [3]:

$$\lambda_0 T_{,ii}-c_\varepsilon\dot{T}-\gamma T\dot{\varepsilon}_{ii}=0. \tag{5}$$

Koshi munosabati [4]:

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), \tag{6}$$

va mos keluvchi boshlang’ich:

$$u_i\big|_{t=t_0}=\phi_i,\ \ \dot{u}_i\big|_{t=t_0}=\psi_i,\ \ T\big|_{t=t_0}=T_0, \tag{7}$$

hamda chegaraviy shartlar:

$$u_i\big|_{\Sigma_1}=u_i^0,\ \ T\big|_{\Sigma_1}=\overline{T}_0,\ \ \sum_{j=1}^{3}\sigma_{ij}n_j\Big|_{\Sigma_2}=S_i^0, \tag{8}$$

bu yerda σ_{ij} – tenzor kuchlanishi, ε_{ij} – deformatsiya tenzori, e_{ij} –, θ – deviator va deformatsiya tenzorining sferik qismlari mos ravishda, σ – kuchlanish tenzorining sferik qismlari, σ_u – kuchlanish tensorlarining intensivligi, ε_u – deformatsiya tenzorlarining intensivligi, δ_{ij} – Kronekker belgisi, u_i – siljish, ρ – zichlik, T – harorat, T_0 – boshlang‘ich harorat, $\vartheta=T-T_0$, c_ε – doimiy deformatsiyadagi issiqlik sig‘imi, λ, μ – Lame elastik konstantalari, λ_0 – issiqlik o‘tkazuvchanlik koeffitsienti, $K=\lambda+\frac{2}{3}\mu$, α – issiqlikdan kengayish koeffitsienti, X_i –, S_i – hajmiy kuchlar va sirt kuchlari, $\gamma=\alpha(3\lambda+2\mu)$. $\sigma_u=\sigma_u(\varepsilon_u,T)$ deformatsiya diagrammasi deb ataladi va har bir harorat T uchun materialning cho‘zilishi yoki buralishi asosida aniqlanadi. Deformatsiya diagrammasini $\sigma_u=\sigma_u(\varepsilon_u)$ qismli chiziqli funksiya sifatida taqdim etish quyidagicha:

$$\sigma_u=2\mu\varepsilon_u+2(\mu-\mu')(\varepsilon_u-\varepsilon_u^*),\quad \varepsilon_u\geq\varepsilon_u^*, \tag{9}$$

(4) va (9) munosabatlarni (2) ga qo‘yib, deformatsion nazariyani ifodalovchi munosabatni quyidagi shaklga keltirish mumkin [4]:

$$\sigma_{ij}=\begin{cases}\lambda\theta\delta_{ij}+2\mu\varepsilon_{ij}-\gamma(T-T_0)\delta_{ij}\ \text{agar}\ \varepsilon_u<\varepsilon_u^*,\\ \lambda\theta\delta_{ij}+2\mu\varepsilon_{ij}-\gamma(T-T_0)\delta_{ij}-2(\mu-\mu')(1-\dfrac{\varepsilon_u^*}{\varepsilon_u})e_{ij}\ \text{agar}\ \varepsilon_u\geq\varepsilon_u^*,\end{cases} \tag{10}$$

bu yerda ε_u^* – elastiklik chegarasi, $\mu^{'}$ – urinma moduli.

3 TO‘G‘RI TO‘RTBURCHAKLI SOHA UCHUN BOG‘LANGAN TERMO-ELASTIK MASALASINI YECHISH USULLARI VA SONLI YECHIMI

Termo-elastiklik holatida (10) ifodani hisobga olgan holda (1)-(6) tenglamalarni quyidagicha yozamiz, ya’ni $\varepsilon_u < \varepsilon_u^*$ ko‘chishlar va T harorat uchun. Bu tenglamalar ikki o‘lchovli holatda quyidagi ko‘rinishga ega bo‘ladi:

$$\begin{aligned}
&(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y}+\mu\frac{\partial^2 u}{\partial y^2}-\gamma\frac{\partial T}{\partial x}+X_1=\rho\frac{\partial^2 u}{\partial t^2},\\
&(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y}+\mu\frac{\partial^2 v}{\partial x^2}-\gamma\frac{\partial T}{\partial y}+X_2=\rho\frac{\partial^2 v}{\partial t^2},
\end{aligned} \tag{11}$$

$$\lambda_0\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-c_\varepsilon\frac{\partial T}{\partial t}-\gamma T\left(\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial^2 v}{\partial y\partial t}\right)=0. \tag{12}$$

boshlang‘ich:

$$u(x,y,t)\big|_{t=0}=\phi_1,\quad \left.\frac{\partial u}{\partial t}\right|_{t=0}=\psi_1,\quad v(x,y,t)\big|_{t=0}=\phi_2,\quad \left.\frac{\partial v}{\partial t}\right|_{t=0}=\psi_2,\; T(x,y,t)\big|_{t=0}=T_0. \tag{13}$$

hamda chegara shartlari:

$$\begin{aligned}
&u(x,y,t)\big|_{x=0}=u_0,\;\; u(x,y,t)\big|_{x=\ell_1}=\overline{u}_0,\;\; u(x,y,t)\big|_{y=0}=u_0',\;\; u(x,y,t)\big|_{y=\ell_2}=\overline{u}_0',\\
&v(x,y,t)\big|_{x=0}=v_0,\;\; v(x,y,t)\big|_{x=\ell_1}=\overline{v}_0,\;\; v(x,y,t)\big|_{y=0}=v_0',\;\; v(x,y,t)\big|_{y=\ell_2}=\overline{v}_0',\\
&T(x,y,t)\big|_{x=0}=T_1(t),\;\; T(x,y,t)\big|_{x=\ell_1}=T_2(t),\;\; T(x,y,t)\big|_{y=0}=T_1'(t),\;\; T(x,y,t)\big|_{y=\ell_2}=T_2'(t).
\end{aligned} \tag{14}$$

$t\ge 0,\;\; 0\le x\le l,\;\; 0\le y\le l$ maydonda uchta parallel to‘g‘ri chiziqlar o‘tkazib, tenglamalardagi hosilalarni chekli-differensial munosabatlar bilan almashtirib $x=ih_1(i=\overline{0,n}),\; y=jh_2(j=\overline{0,n}),\; t=k\tau\;(k=0,1,2,...)$ (11)-(14) tenglamalardan quyidagilarni topishimiz mumkin:

$$\begin{aligned}
&(\lambda+2\mu)\frac{u^k_{i+1,j}-2u^k_{i,j}+u^k_{i-1,j}}{h_1^2}+(\lambda+\mu)\frac{v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1}}{4h_1h_2}+\mu\frac{u^k_{i,j+1}-2u^k_{i,j}+u^k_{i,j-1}}{h_2^2}-\gamma\frac{T^k_{i+1,j}-T^k_{i-1,j}}{2h_1}=\rho\frac{u^{k+1}_{i,j}-2u^k_{i,j}+u^{k-1}_{i,j}}{\tau^2},\\
&(\lambda+2\mu)\frac{v^k_{i,j+1}-2v^k_{i,j}+v^k_{i,j-1}}{h_2^2}+(\lambda+\mu)\frac{u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1}}{4h_1h_2}+\mu\frac{v^k_{i+1,j}-2v^k_{i,j}+v^k_{i-1,j}}{h_1^2}-\gamma\frac{T^k_{i,j+1}-T^k_{i,j-1}}{2h_2}=\rho\frac{v^{k+1}_{i,j}-2v^k_{i,j}+v^{k-1}_{i,j}}{\tau^2},
\end{aligned} \tag{15}$$

$$\lambda_0\left(\frac{T^k_{i+1,j}-Tu^k_{i,j}+T^k_{i-1,j}}{h_1^2}+\frac{T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1}}{h_2^2}\right)-c_\varepsilon\frac{T^{k+1}_{i,j}-T^k_{i,j}}{\tau}-\gamma T^k_{i,j}\left(\frac{u^{k+1}_{i+1,j}-u^{k+1}_{i-1,j}-u^{k-1}_{i+1,j}+u^{k-1}_{i-1,j}}{4h_1\tau}+\frac{v^{k+1}_{i,j+1}-v^{k+1}_{i,j-1}-v^{k-1}_{i,j+1}+v^{k-1}_{i,j-1}}{4h_2\tau}\right)=0. \tag{16}$$

(13) boshlang‘ich shartlardagi hosilalarni chekli-ayirmali munosabatlar bilan almashtirib, quyidagi munosabatni topamiz,

$$\begin{aligned}
&u^0_{ij}=\phi_1,\;\; v^0_{ij}=\phi_2,\\
&\frac{u^1_{i,j}-u^{-1}_{i,j}}{2\tau}=\psi_1(x_i,y_j)\;\text{ yoki }\; u^1_{i,j}=2\tau\psi_1(x_i,y_j)+u^{-1}_{i,j},\\
&\frac{v^1_{i,j}-v^{-1}_{i,j}}{2\tau}=\psi_2(x_i,y_j)\;\text{ yoki }\; v^1_{i,j}=2\tau\psi_2(x_i,y_j)+v^{-1}_{i,j}.
\end{aligned} \tag{17}$$

$k=0$ bo‘lganda (15) tenglamadan birinchi qatlamni hisoblab olamiz, hamda $u^{-1}_{i,j},\, v^{-1}_{i,j}$ larni (17) munosabatdan topib olamiz.

$$\begin{aligned}
&u^1_{i,j}=\frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u^0_{i+1,j}-2u^0_{i,j}+u^0_{i-1,j}}{h_1^2}+\mu\frac{u^0_{i,j+1}-2u^0_{i,j}+u^0_{i,j-1}}{h_2^2}+(\lambda+\mu)\frac{v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1}}{4h_1h_2}-\gamma\frac{T^0_{i+1,j}-T^0_{i-1,j}}{2h_1}\right)+2u^0_{i,j}+2\tau\psi_1\right),\\
&v^1_{i,j}=\frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v^0_{i,j+1}-2v^0_{i,j}+u^0_{i,j-1}}{h_2^2}+\mu\frac{v^0_{i+1,j}-2v^0_{i,j}+v^0_{i-1,j}}{h_1^2}+(\lambda+\mu)\frac{u^0_{i+1,j+1}-u^0_{i-1,j+1}-u^0_{i+1,j-1}+u^0_{i-1,j-1}}{4h_1h_2}-\gamma\frac{T^0_{i,j+1}-T^0_{i,j-1}}{2h_2}\right)+2v^0_{i,j}+2\tau\psi_2\right),
\end{aligned} \tag{18}$$

$$T_{i,j}^{1} = \frac{\tau}{c_{\varepsilon}}\left(\lambda_0\left(\frac{T_{i+1,j}^{0}-2T_{i,j}^{0}+T_{i-1,j}^{0}}{h_1^2}+\frac{T_{i,j+1}^{0}-2T_{i,j}^{0}+T_{i,j-1}^{0}}{h_2^2}\right)-\right.$$
$$\left.-\gamma T_{i,j}^{0}\left(\frac{u_{i+1,j}^{1}-u_{i-1,j}^{1}-u_{i+1,j}^{0}+u_{i-1,j}^{0}}{2h_1\tau}+\frac{v_{i,j+1}^{1}-v_{i,j-1}^{1}-v_{i,j+1}^{0}+v_{i,j-1}^{0}}{2h_2\tau}\right)\right)+T_{i,j}^{0}. \tag{19}$$

Boshlang‘ich (13) va chegaraviy (14) shartlarga ko‘ra, izlanayotgan funksiyalarning qiymatlari $u_{i,j}^{k}$, $v_{i,j}^{k}$, $T_{i,j}^{k}$ ikkita dastlabki qatlamda ma’lum $k=0,1$ va ko‘rib chiqilayotgan sohaning chegaralarida qiymatlari mavjud. Bu funksiyalarning qolgan qatlamlardagi qiymatlarini, ya’ni $k=2,3,...$ quyidagi rekurrent munosabatlari orqali topish mumkin:

$$u_{i,j}^{k+1} = \frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u_{i+1,j}^{k}-2u_{i,j}^{k}+u_{i-1,j}^{k}}{h_1^2}+\mu\frac{u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k}}{h_2^2}+\right.$$
$$\left.+(\lambda+\mu)\frac{v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k}}{4h_1h_2}-\gamma\frac{T_{i+1,j}^{k}-T_{i-1,j}^{k}}{2h_1}\right)+2u_{i,j}^{k}-u_{i,j}^{k-1}, \tag{20}$$

$$v_{i,j}^{k+1} = \frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v_{i,j+1}^{k}-2v_{i,j}^{k}+u_{i,j-1}^{k}}{h_2^2}+\mu\frac{v_{i+1,j}^{k}-2v_{i,j}^{k}+v_{i-1,j}^{k}}{h_1^2}+\right.$$
$$\left.+(\lambda+\mu)\frac{u_{i+1,j+1}^{k}-u_{i-1,j+1}^{k}-u_{i+1,j-1}^{k}+u_{i-1,j-1}^{k}}{4h_1h_2}-\gamma\frac{T_{i,j+1}^{k}-T_{i,j-1}^{k}}{2h_2}\right)+2v_{i,j}^{k}-v_{i,j}^{k-1}, \tag{21}$$

$$T_{i,j}^{k+1} = \frac{\tau}{c_{\varepsilon}}\left(\lambda_0\left(\frac{T_{i+1,j}^{k}-T_{i,j}^{k}+T_{i-1,j}^{k}}{h_1^2}+\frac{T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}}{h_2^2}\right)-\right.$$
$$\left.-\gamma T_{i,j}^{k}\left(\frac{u_{i+1,j}^{k+1}-u_{i-1,j}^{k+1}-u_{i+1,j}^{k-1}+u_{i-1,j}^{k-1}}{4h_1\tau}+\frac{v_{i,j+1}^{k+1}-v_{i,j-1}^{k+1}-v_{i,j+1}^{k-1}+v_{i,j-1}^{k-1}}{4h_2\tau}\right)\right)+T_{i,j}^{k}. \tag{22}$$

(15-16) tenglamalarni mos ravishda $u_{i,j}^{k+1}$, $v_{i,j}^{k+1}$, $T_{i,j}^{k+1}$ ga nisbatan rekkurent formulalar bilan yechish orqali topilgan. Rekurrent formulalarga keltirilishi mumkin bo‘lgan chekli-ayirmali tenglamalar odatda oshkor sxemalar deb ataladi hamda koordinatalar va vaqt bo‘yicha panjara uzunligi bo‘yicha cheklovga ega.

Ko‘rsatilgan cheklovlarsiz oshkormas sxemalar deb ataladigan boshqa turdagi chekli-ayirmali tenglamalar tuzilishi mumkin. Buning uchun (15-16) chekli-ayirmali tenglamalarning birinchi hadlaridagi k indeksni k+1 indeks bilan almashtirish kerak, ya’ni [6, 8, 15]:

$$(\lambda+2\mu)\frac{u_{i+1,j}^{k+1}-2u_{i,j}^{k+1}+u_{i-1,j}^{k+1}}{h_1^2}+(\lambda+\mu)\frac{v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k}}{4h_1h_2}+\mu\frac{u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k}}{h_2^2}-\gamma\frac{T_{i+1,j}^{k}-T_{i-1,j}^{k}}{2h_1}=\rho\frac{u_{i,j}^{k+1}-2u_{i,j}^{k}+u_{i,j}^{k-1}}{\tau^2},$$
$$(\lambda+2\mu)\frac{v_{i,j+1}^{k+1}-2v_{i,j}^{k+1}+v_{i,j-1}^{k+1}}{h_2^2}+(\lambda+\mu)\frac{u_{i+1,j+1}^{k}-u_{i-1,j+1}^{k}-u_{i+1,j-1}^{k}+u_{i-1,j-1}^{k}}{4h_1h_2}+\mu\frac{v_{i+1,j}^{k}-2v_{i,j}^{k}+v_{i-1,j}^{k}}{h_1^2}-\gamma\frac{T_{i,j+1}^{k}-T_{i,j-1}^{k}}{2h_2}=\rho\frac{v_{i,j}^{k+1}-2v_{i,j}^{k}+v_{i,j}^{k-1}}{\tau^2}, \tag{23}$$

$$\lambda_0\left(\frac{T_{i+1,j}^{k+1}-2T_{i,j}^{k+1}+T_{i-1,j}^{k+1}}{h_1^2}+\frac{T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}}{h_2^2}\right)-c_{\varepsilon}\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\tau}-\gamma T_{i,j}^{k}\left(\frac{u_{i+1,j}^{k+1}-u_{i-1,j}^{k+1}-u_{i+1,j}^{k-1}+u_{i-1,j}^{k-1}}{4h_1\tau}+\frac{v_{i,j+1}^{k+1}-v_{i,j-1}^{k+1}-v_{i,j+1}^{k-1}+v_{i,j-1}^{k-1}}{4h_2\tau}\right)=0. \tag{24}$$

(23) tenglamada oldidagi koeffitsientlarni bildiruvchi $u_{i+1,j}^{k+1}$, u_{ij}^{k+1} va $u_{i-1,j}^{k+1}$ kabi a_i, b_i va c_i mos ravishda va qolgan barchasini esa f_{ij} kabi belgilab, quyidagi ko‘rinishda yozish mumkin:

$$a_i u_{i+1,j}^{k+1}+b_i u_{i,j}^{k+1}+c_i u_{i-1,j}^{k+1}=f_{ij}, \tag{25}$$

bu yerda

$$a_i=\frac{\lambda+2\mu}{h_1^2},\quad b_i=-\frac{2(\lambda+2\mu)}{h_1^2}-\frac{\rho}{\tau^2},\quad c_i=\frac{\lambda+2\mu}{h_1^2},$$

$$f_{ij}=\gamma\frac{T_{i+1,j}^{k}-T_{i-1,j}^{k}}{2h_1}+\rho\frac{u_{i,j}^{k-1}-2u_{i,j}^{k}}{\tau^2}-(\lambda+\mu)\frac{v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k}}{4h_1h_2}-\mu\frac{u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k}}{h_2^2}.$$

Ma'lumki (25) ko'rinishdagi chekli-ayirmali tenglamalar uch dioganal matritsali algebraik tenglamalar sistemasi bo'lib, $k = 2,3....$ ning har bir qiymat uchun haydash usuli bilan yechilishi mumkin. Oshkormas sxemaning yaqinlashishi uchun diagonal ustunlik sharti bajarilishi kerak [26] ya'ni. $|b_i| \geq |a_i| + |c_i|$. Xuddi shunday tarzda (23) tenglamani $v_{i,j}^k$ ga nisbatan quyidagi shaklda yozish mumkin [26]:

$$a_i v_{i,j+1}^{k+1} + b_i v_{i,j}^{k+1} + c_i v_{i,j-1}^{k+1} = f_{ij}, \tag{26}$$

bu yerda

$$a_i = \frac{\lambda + 2\mu}{h_2^2}, \quad b_i = -\frac{2(\lambda + 2\mu)}{h_2^2} - \frac{\rho}{\tau^2}, \quad c_i = \frac{\lambda + 2\mu}{h_2^2},$$

$$f_{ij} = \gamma \frac{T_{i,j+1}^k - T_{i,j-1}^k}{2h_2} + \rho \frac{v_{i,j}^{k-1} - 2v_{i,j}^k}{\tau^2} - \mu \frac{v_{i+1,j}^k - 2v_{i,j}^k + v_{i-1,j}^k}{h_1^2} - (\lambda + \mu) \frac{u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k}{4h_1 h_2}.$$

Xuddi shunday, (24) differensial tenglamani $T_{i,j}^k$ ga nisbatan tegishli tenglamaga keltirilishi mumkin:

$$a_i T_{i+1,j}^{k+1} + b_i T_{i,j}^{k+1} + c_i T_{i-1,j}^{k+1} = f_{ij}, \tag{27}$$

bu yerda

$$a_i = \frac{\lambda_0}{h_1^2}, \quad b_i = -\frac{2\lambda_0}{h_1^2} - \frac{C_\varepsilon}{\tau}, \quad c_i = \frac{\lambda_0}{h_1^2},$$

$$f_{ij} = \gamma T_{ij}^k \left(\frac{u_{i+1,j}^{k+1} - u_{i-1,j}^{k+1} - u_{i+1,j}^{k-1} + u_{i-1,j}^{k-1}}{4h_1\tau} + \frac{v_{i,j+1}^{k+1} - v_{i,j-1}^{k+1} - v_{i,j+1}^{k-1} + v_{i,j-1}^{k-1}}{4h_2\tau} \right) - C_\varepsilon \frac{T_{ij}^k}{\tau} - \lambda_0 \frac{T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k}{h_2^2}.$$

Temperatura maydonida qattiq holatni o'rganishga alohida e'tibor beriladi. Shuning uchun misol tariqasida boshlang'ich va chegaraviy shartlari nolga teng bo'lgan harorat maydoni ta'sirida hamma tomondan siqilgan to'g'ri to'rtburchakni qaraymiz. Bunda boshlang'ich va chegaraviy shartlarning diskret analoglari quyidagi ko'rinishga ega bo'ladi:

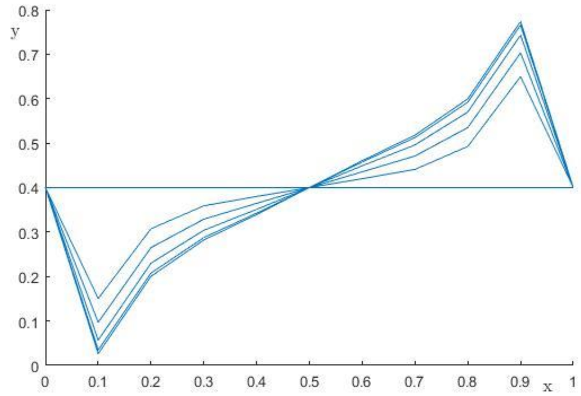
$$u_{ij}^0 = 0, \quad \frac{u_{ij}^1 - u_{ij}^0}{\tau} = 0, \quad v_{ij}^0 = 0, \quad \frac{v_{ij}^1 - v_{ij}^0}{\tau} = 0, \; T_{ij}^0 = T_0 + T_0 \sin\left(\frac{\pi x_i}{l_1}\right) \sin\left(\frac{\pi y_j}{l_2}\right),$$

$$u_{0j}^k = 0, \quad u_{N_1 j}^k = 0, \quad u_{i0}^k = 0, \quad u_{iN_2}^k = 0, \quad v_{0j}^k = 0, \quad v_{N_1 j}^k = 0, \quad v_{i0}^k = 0, \quad v_{iN_2}^k = 0,$$

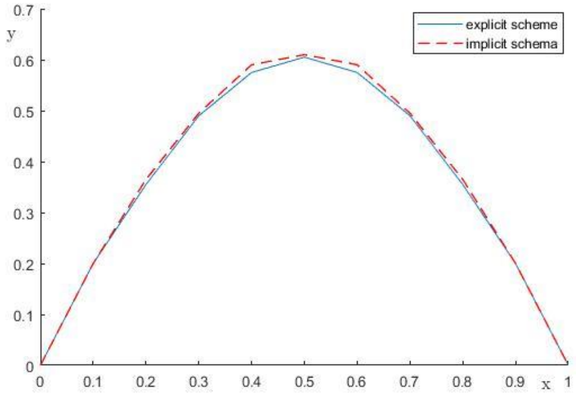
$$T_{0j}^k = 0, \quad T_{N_1 j}^k = 0, \quad T_{i0}^k = 0, \quad T_{iN_2}^k = 0.$$

Boshlang'ich konstantalar sifatida quyidagi qiymatlardan foydalanilgan.

$$\lambda = 0.78 * 10^5 kg / sm^2, \; \mu = 0.7 * 10^5 kg / sm^2, \; \alpha = 0.05 * 10^{-5}, \; \rho = 0.86 * 10^4 \; kg / m^3, \; \lambda_0 = 0.06,$$
$$c_\varepsilon = 3.4 * 10^4 J / (kg * K), \; T_0 = 15^0 C, \quad h_1 = 0.1, \; h_2 = 0.1, \; \tau = 0.01, \; \ell_i = 1, \; n_1 = n_2 = 10.$$



1-rasm. *u(x,y,t)* funksiyaning *t*=0.9 da *OX* o'qi bo'yicha taqsimot grafigi (oshkor sxema) $\varepsilon = 0.001$



2-rasm. *v(x,y,t) y*=0.6, *t*=0.9 tugun nuqtasida *OX* o‘qi bo‘ylab ko‘chish komponentasining o‘zgarishi

Ikki usul bilan olingan sonli natijalar bir-biriga juda yaqin ekanligiga, geometrik tasvirlar esa bir-biriga mos kelishi, ya’ni deyarli bir xil ekanligiga ishonch hosil qilish mumkin. Termoelastiklik va issiqlik o‘tkazuvchanlik tenglamalarini birgalikda yechish qattiq jismlarning mexanik va issiqlik ta’sirida chiziqli va nochiziqli deformatsiyalanish jarayonini yanada adekvat tavsiflash imkonini beradi.

4 IKKI O‘LCHAMLI BOG‘LANGAN TERMO-PLASTIK MASALANING SONLI YECHIMI

Termo-plastiklikning bog‘langan chegaraviy masalasi (1-10) tenglamalarni hisobga olgan holda $\varepsilon_u \geq \varepsilon_u^*$ uchun ko‘chishlar va haroratga nisbatan yozilish mumkin, bu ikki o‘lchovli holatda quyidagi ko‘rinishga ega:

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y}+\mu\frac{\partial^2 u}{\partial y^2}-\gamma\frac{\partial T}{\partial x}+X_1^*=\rho\frac{\partial^2 u}{\partial t^2},$$
$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y}+\mu\frac{\partial^2 v}{\partial x^2}-\gamma\frac{\partial T}{\partial y}+X_2^*=\rho\frac{\partial^2 v}{\partial t^2}. \quad (28)$$

$$\lambda_0\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-c_\varepsilon\frac{\partial T}{\partial t}-\gamma T\left(\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial^2 v}{\partial y\partial t}\right)=0, \quad (29)$$

boshlang‘ich:

$$u(x,y,t)\big|_{t=0}=\phi_1,\ \ \frac{\partial u}{\partial t}\bigg|_{t=0}=\psi_1,\ \ v(x,y,t)\big|_{t=0}=\phi_2,\ \ \frac{\partial v}{\partial t}\bigg|_{t=0}=\psi_2,\ T(x,y,t)\big|_{t=0}=T_0. \quad (30)$$

hamda chegaraviy shartlar:

$$u(x,y,t)\big|_{x=0}=u_0,\ u(x,y,t)\big|_{x=\ell_1}=\bar{u}_0,\ u(x,y,t)\big|_{y=0}=u_0',\ u(x,y,t)\big|_{y=\ell_2}=\bar{u}_0',$$
$$v(x,y,t)\big|_{x=0}=v_0,\ v(x,y,t)\big|_{x=\ell_1}=\bar{v}_0,\ v(x,y,t)\big|_{y=0}=v_0',\ v(x,y,t)\big|_{y=\ell_2}=\bar{v}_0', \quad (31)$$
$$T(x,y,t)\big|_{x=0}=T_1(t),\ T(x,y,t)\big|_{x=\ell_1}=T_2(t),\ T(x,y,t)\big|_{y=0}=T_1'(t),\ T(x,y,t)\big|_{y=\ell_2}=T_2'(t),$$

bu yerda:

$$X_1^*=(-\frac{4}{3}\frac{\partial^2 u}{\partial x^2}-\frac{1}{3}\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2})(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u}),$$
$$X_2^*=(-\frac{4}{3}\frac{\partial^2 v}{\partial y^2}-\frac{1}{3}\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial^2 v}{\partial x^2})(\mu-\mu')(1-\frac{\varepsilon_u^*}{\varepsilon_u}). \quad (32)$$

$t\geq 0,\ \ 0\leq x\leq l,\ \ 0\leq y\leq l$ sohada berilgan $x=ih_1(i=\overline{0,n}),\ y=jh_2(j=\overline{0,n}),\ \ t=k\tau(k=0,1,2,...)$ parallel to‘g‘ri chiziqlarni qarasak va (28)–(31) tenglamalardagi hosilalarni chekli-ayirmali munosabatlar bilan almashtirsak,

$$(\lambda+2\mu)\frac{u_{i+1,j}^{k}-2u_{i,j}^{k}+u_{i-1,j}^{k}}{h_1^2}+(\lambda+\mu)\frac{v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k}}{4h_1h_2}+\mu\frac{u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k}}{h_2^2}-\gamma\frac{T_{i+1,j}^{k}-T_{i-1,j}^{k}}{2h_1}+X_1^*=\rho\frac{u_{i,j}^{k+1}-2u_{i,j}^{k}+u_{i,j}^{k-1}}{\tau^2},$$

$$(\lambda+2\mu)\frac{v_{i,j+1}^{k}-2v_{i,j}^{k}+v_{i,j-1}^{k}}{h_2^2}+(\lambda+\mu)\frac{u_{i+1,j+1}^{k}-u_{i-1,j+1}^{k}-u_{i+1,j-1}^{k}+u_{i-1,j-1}^{k}}{4h_1h_2}+\mu\frac{v_{i+1,j}^{k}-2v_{i,j}^{k}+v_{i-1,j}^{k}}{h_1^2}-\gamma\frac{T_{i,j+1}^{k}-T_{i,j-1}^{k}}{2h_2}+X_2^*=\rho\frac{v_{i,j}^{k+1}-2v_{i,j}^{k}+v_{i,j}^{k-1}}{\tau^2}, \quad (33)$$

differensial tenglamalarni mos ravishda $u_{i,j}^{k+1}$, $v_{i,j}^{k+1}$, $T_{i,j}^{k+1}$ ga nisbatan rekkurent formula orqali yechamiz:

$$u_{i,j}^{k+1}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u_{i+1,j}^{k}-2u_{i,j}^{k}+u_{i-1,j}^{k}}{h_1^2}+\mu\frac{u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k}}{h_2^2}+\right.$$
$$\left.+(\lambda+\mu)\frac{v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k}}{4h_1h_2}-\gamma\frac{T_{i+1,j}^{k}-T_{i-1,j}^{k}}{2h_1}+X_1^*\right)+2u_{i,j}^{k}-u_{i,j}^{k-1}, \quad (35)$$

$$v_{i,j}^{k+1}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v_{i,j+1}^{k}-2v_{i,j}^{k}+u_{i,j-1}^{k}}{h_2^2}+\mu\frac{v_{i+1,j}^{k}-2v_{i,j}^{k}+v_{i-1,j}^{k}}{h_1^2}+\right.$$
$$\left.+(\lambda+\mu)\frac{u_{i+1,j+1}^{k}-u_{i-1,j+1}^{k}-u_{i+1,j-1}^{k}+u_{i-1,j-1}^{k}}{4h_1h_2}-\gamma\frac{T_{i,j+1}^{k}-T_{i,j-1}^{k}}{2h_2}+X_2^*\right)+2v_{i,j}^{k}-v_{i,j}^{k-1}, \quad (36)$$

$$T_{i,j}^{k+1}=\frac{\tau}{c_\varepsilon}\left(\lambda_0\left(\frac{T_{i+1,j}^{k}-Tu_{i,j}^{k}+T_{i-1,j}^{k}}{h_1^2}+\frac{T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}}{h_2^2}\right)-\right.$$
$$\left.-\gamma T_{i,j}^{k}\left(\frac{u_{i+1,j}^{k+1}-u_{i-1,j}^{k+1}-u_{i+1,j}^{k-1}+u_{i-1,j}^{k-1}}{4h_1\tau}+\frac{v_{i,j+1}^{k+1}-v_{i,j-1}^{k+1}-v_{i,j+1}^{k-1}+v_{i,j-1}^{k-1}}{4h_2\tau}\right)\right)+T_{i,j}^{k}. \quad (37)$$

Boshlang‘ich (30) va chegaraviy (31) shartlarga ko‘ra tugun funksiyalarning $u_{i,j}^{k}$, $v_{i,j}^{k}$ va $T_{i,j}^{k}$ qiymatlari boshlang‘ich ikki qatlamda k=0 va k=1 ma’lum. U holda k=0 va k=1 bo‘lganda (35-37) tenglamalardan (30) boshlang‘ich shartlarning chekli-ayirmali analoglarini hisobga olgan holda $\varepsilon_u \le \varepsilon_u^*$ uchun quyidagi ifodalarni topish mumkin:

$$u_{i,j}^{1}=\frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u_{i+1,j}^{0}-2u_{i,j}^{0}+u_{i-1,j}^{0}}{h_1^2}+\mu\frac{u_{i,j+1}^{0}-2u_{i,j}^{0}+u_{i,j-1}^{0}}{h_2^2}+\right.\right.$$
$$\left.\left.+(\lambda+\mu)\frac{v_{i+1,j+1}^{0}-v_{i-1,j+1}^{0}-v_{i+1,j-1}^{0}+v_{i-1,j-1}^{0}}{4h_1h_2}-\gamma\frac{T_{i+1,j}^{0}-T_{i-1,j}^{0}}{2h_1}\right)+2u_{i,j}^{0}+2\tau\psi_1\right), \quad (38)$$

$$v_{i,j}^{1}=\frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v_{i,j+1}^{0}-2v_{i,j}^{0}+u_{i,j-1}^{0}}{h_2^2}+\mu\frac{v_{i+1,j}^{0}-2v_{i,j}^{0}+v_{i-1,j}^{0}}{h_1^2}+\right.\right.$$
$$\left.\left.+(\lambda+\mu)\frac{u_{i+1,j+1}^{0}-u_{i-1,j+1}^{0}-u_{i+1,j-1}^{0}+u_{i-1,j-1}^{0}}{4h_1h_2}-\gamma\frac{T_{i,j+1}^{0}-T_{i,j-1}^{0}}{2h_2}\right)+2v_{i,j}^{0}+2\tau\psi_2\right), \quad (39)$$

$$T_{i,j}^{1}=\frac{\tau}{c_\varepsilon}\left(\lambda_0\left(\frac{T_{i+1,j}^{0}-2T_{i,j}^{0}+T_{i-1,j}^{0}}{h_1^2}+\frac{T_{i,j+1}^{0}-2T_{i,j}^{0}+T_{i,j-1}^{0}}{h_2^2}\right)-\right.$$
$$\left.-\gamma T_{i,j}^{0}\left(\frac{u_{i+1,j}^{1}-u_{i-1,j}^{1}-u_{i+1,j}^{0}+u_{i-1,j}^{0}}{2h_1\tau}+\frac{v_{i,j+1}^{1}-v_{i,j-1}^{1}-v_{i,j+1}^{0}+v_{i,j-1}^{0}}{2h_2\tau}\right)\right)+T_{i,j}^{0}. \quad (40)$$

Shunday qilib, (33-34) chekli-ayirmali tenglamalar oshkor sxemalarni ifodalaydi va quyidagi termomexanik boshlang‘ich shartlar bilan (35-40) rekkurent munosabatlar yordamida yechish mumkin:

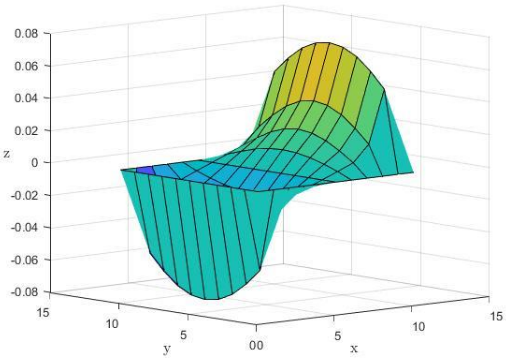
$$u_{ij}^{0}=0,\quad \frac{u_{ij}^{1}-u_{ij}^{0}}{\tau}=0,\quad v_{ij}^{0}=0,\quad \frac{v_{ij}^{1}-v_{ij}^{0}}{\tau}=0,$$
$$T_{ij}^{0}=T_0+T_0\sin\left(\frac{\pi x_i}{l_1}\right)\sin\left(\frac{\pi y_j}{l_2}\right),$$

va chegara shartlari:

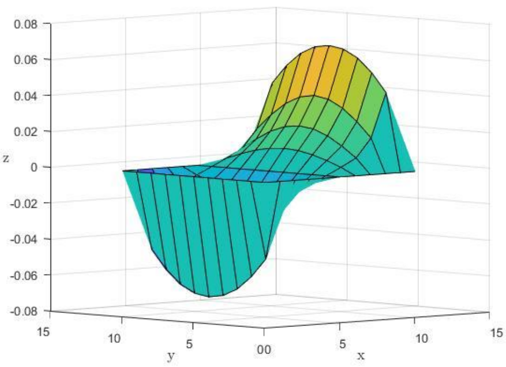
$$u_{0j}^k = 0,\quad u_{N_1 j}^k = 0,\quad u_{i0}^k = 0,\quad u_{iN_2}^k = 0,$$
$$v_{0j}^k = 0,\quad v_{N_1 j}^k = 0,\quad v_{i0}^k = 0,\quad v_{iN_2}^k = 0,$$
$$T_{0j}^k = 0,\quad T_{N_1 j}^k = 0,\quad T_{i0}^k = 0,\quad T_{iN_2}^k = 0.$$

Elastik-plastik termo-mexanik konstantalar quyidagi qiymatlarga ega:

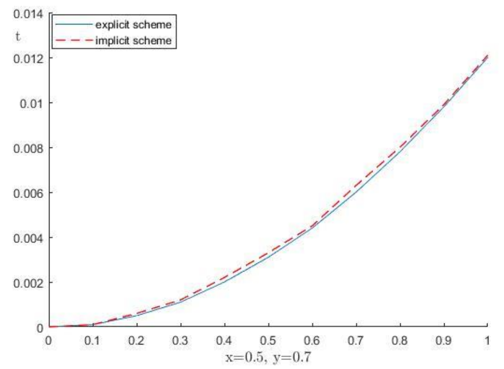
$$\lambda = 0.78*10^5 kg/sm^2,\ \alpha = 0.05*10^{-5},\ \mu = 0.7*10^5 kg/sm^2,\ \ \rho = 0.86*10^4\ kg/m^3,$$
$$c_\varepsilon = 3.4*10^4 J/(kg*K),\ T_0 = 15^0 C,\ \lambda_0 = 0.06,\ \mu' = 0.4*10^5 kg/sm^2,\ h_1 = 0.1,\ h_2 = 0.1,\ \tau = 0.01,$$
$$\ell_i = 1,\ n_1 = n_2 = 10.$$



3-rasm. *u(x,y,t)* funksiyaning *OX* o‘qi bo‘ylab *t*=0.9 dagi taqsimot grafigi (oshkor sxema) $\varepsilon = 0.001$



4-rasm. *u(x,y,t)* funksiyaning *OX* o‘qi bo‘ylab *t*=0.9 dagi taqsimot grafigi (oshkormas sxema) $\varepsilon = 0.001$



5-rasm. *u(x,y,t)* funksiyaning *OZ* o‘qi bo‘ylab *x*=0.5, *y*=0.7 tugun nuqtasida ko‘chishlarning o‘zgarishi (oshkor va oshkormas sxemalar)

5 XULOSA

Bog'langan dinamik termo-elastiklik va termo-plastiklik chegaraviy masalalari qarab chiqilgan. Bog'langan termo-plastiklik masalaasi Ilyushinning deformatsion plastiklikning nazariyasiga asoslanadi. Turli boshlang'ich va chegaraviy shartlarda to'rtburchaklar uchun termo-elastik-plastik chegaraviy masalalar qaraladi. Diskret tenglamalar oshkor va oshkormas sxemalar ko'rinishida chekli-ayirmalar usuli bilan tuziladi. Oshkor sxemalarning yechimi ko'chish komponentlari va harorat bilan bog'liq takrorlanish munosabatlariga keltirilgan. Oshkormas sxemalar mos keladigan yo'nalishlar bo'yicha haydash usuli yordamida sonli yechilgan.

Ikki o'lchovli termo-elastik-plastik chegaraviy masalalarni yechish uchun samarali sonli algoritm va tegishli dasturiy ta'minot ishlab chiqilgan. Berilgan harorat maydoniga ega bo'lgan, har tomondan siqilgan to'g'ri to'rtburchakda bir qator termo-elastik-plastik masalalar yechilgan. Turli usullar bilan olingan sonli natijalar taqqoslangan va yaqin moslik olingan. Harorat maydonining ko'chish taqsimotiga, shuningdek, to'g'ri to'rtburchakdagi plastik zonalarning paydo bo'lishiga ta'siri o'rganilgan.

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NUMERICAL SOLUTION OF PROBLEMS IN THE DYNAMICALLY COUPLED TWO-DIMENSIONAL THEORY OF THERMOELASTICITY AND THERMOPLASTICITY

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Abstract. The article considers a numerical method for solving a two-dimensional coupled dynamic thermoplastic boundary value problem based on deformation theory of plasticity. Discrete equations are compiled by the finite-difference method in the form of explicit and implicit schemes. The solution of the explicit schemes is reduced to the recurrence relations regarding the components of displacement and temperature. Implicit schemes are efficiently solved using the elimination method for systems with a three-diagonal matrix along the appropriate directions. In this case, the diagonal predominance of the transition matrices ensures the convergence of implicit difference schemes. The problem of a thermoplastic rectangle clamped from all sides under the action of an internal thermal field is solved numerically. The stress-strain state of a thermoplastic rectangle and the distribution of displacement and temperature over various sections and points in time have been investigated.

Keywords: thermoplasticity, displacement, temperature, stress, differential equation, explicit scheme, convergence.

ЧИСЛЕННОЕ РЕШЕНИЕ ЗАДАЧ ДИНАМИЧЕСКИ СВЯЗАННОЙ ДВУМЕРНОЙ ТЕОРИИ ТЕРМОУПРУГОСТИ И ТЕРМОПЛАСТИЧНОСТИ

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Аннотация. В статье рассмотрен численный метод решения двухмерной связанной динамической краевой задачи термопластических по деформационной теории изотропных тел. Дискретные уравнения составлены конечно-разностным методом в виде явных и неявных схем. Решение явных схем приведены к рекуррентным соотношениями относительно компонентам перемещений и температуры. Неявные схемы, эффективным образом, приведены к последовательному применению метода прогонки по соответствующим направлениям. При этом, диагональным преобладанием матриц перехода обеспечивает сходимость неявных разностных схем. Решена численно задача о термопластическом защемленном со всех сторон прямоугольнике находящегося под действием внутреннего теплового поля. Исследовано напряженно-деформированное состояние термопластического прямоугольника и распределение перемещения и температуры по различным сечениям и моментах времени.

Ключевые слова: термопластичность, перемещение, температура, напряжение, разностное уравнение, явная схема, сходимость.