UDC 516.9

REVIEW OF RESEARCH ON MATHEMATICAL MODELING OF THE PROCESS OF IN-SITU LEACHING OF MINERAL RESOURCES

Ravshanov N.[1], [+] *Usmonov L.*[1]

[1] Digital technologies and artificial intelligence development research institute,
Tashkent, Uzbekistan

[+] uslochinbek@gmail.com

Abstract. The paper presents a summary of the work done on the application of underground mining methods in the backfilling of ore deposits. In addition, basic information for future research is provided. The analysis of the numerical calculations showed that at the initial stage of ore deposit development, the active reaction of the reagent supplied by injection wells occurs around the wells, and as shown by numerical calculations in the computer system, this area expands over time depending on the permeability and porosity of the ore deposit. In this study, an extensive literature review and gap analysis are used to identify the limitations and opportunities for further research on the in-situ leaching (ISL) method for ore mining for integrated resource development planning. Moreover, our goal is to improve the methods of mining various minerals using the underground leaching method and to create a mathematical model for it based on the work performed. We believe that the extraction of minerals, especially ores, has always been and will be a pressing economic and political issue. Therefore, the demand for research in this area will continue to grow.

Keywords: In-situ leaching, mathematical model, mining process, modeling, uranium deposit, Darcy’s law.

1 INTRODUCTION

It is important that a strategic mine plan makes optimum use of the available resources and provides a consistent supply of quality ore to ensure sustainable production and profitability. This requires the development of a well-integrated strategy of underground mining options and their interactions. Understanding the current tools and methodologies used in the mining industry for underground mining options and transition planning is essential to address complex and deep deposits that are amenable to underground mining.

Fundamental scientific research aimed at developing and improving the methodology of mathematical modeling of complex heat and mass transfer processes based on high-performance computer computing systems is very relevant and is carried out in many leading scientific centers and higher educational institutions around the world.

It is known that modern mining science today pays special attention to the development of new technological methods for the extraction of minerals [1-7]. This is natural, because the development of society leads to an increase in the material and technical base of the national economy and the demand for their rational use. The processed deposits are located underground in conditions of strong waterlogging, consist of wet sandy rocks and, of course, very poor ores, and the efficiency of extraction depends only on the use of advanced technological processes and high-level mining operations.

The development of deposits by traditional methods has very limited possibilities. In recent years, extensive research has been conducted to process such deposits by geological methods. It is known that physicochemical, diffusion hydrodynamic processes occur in the extraction of useful elements from the earth's crust. Geotechnical methods used to control processes the underground mixing method is of particular importance because it does not have a negative impact on environmental protection, since its use does not harm the surface and air layers at all.

In-Situ leaching (ISL) is a method of mineral extraction in permeable media, which involves constructing a network of injection and production wells. Through this network, a solution such as sulfuric acid is pumped into the strata to dissolve solid minerals, enabling extraction. The process is controlled by managing the flow rate and the concentration of the injected solution, and the resulting product is extracted through well filters. This article reviews the work done on mathematical modeling of the hydrodynamic processes of ISL during ore deposit development. It emphasizes the importance of a strategic mining plan to ensure optimal resource utilization, sustainable mining practices, and profitability through a consistent supply of high-quality ore.

Developing a well-integrated extraction strategy for underground mines and their interactions is crucial. Understanding current tools and methodologies for planning transitions in underground mining is critical for handling complex and deeply situated ore bodies. This research includes an extensive literature review and analysis of gaps, which can inform further studies on optimizing transitions and opportunities in underground mining to enhance comprehensive resource development planning.

This article analyzes the work of a number of scientists on mathematical models (MM) of ISL in the process of ore deposit development. During these studies, several methods were used to solve the problem numerically. For example, stream mining, iteration, etc. In addition, studies on modeling the removal of pollutants from the production area after the mining process are also considered. A device that injects liquid into the ground and removes the mixture formed underground. The technical classification of wells that inject fluid underground and extract the mixture formed underground is presented in the figure 1.

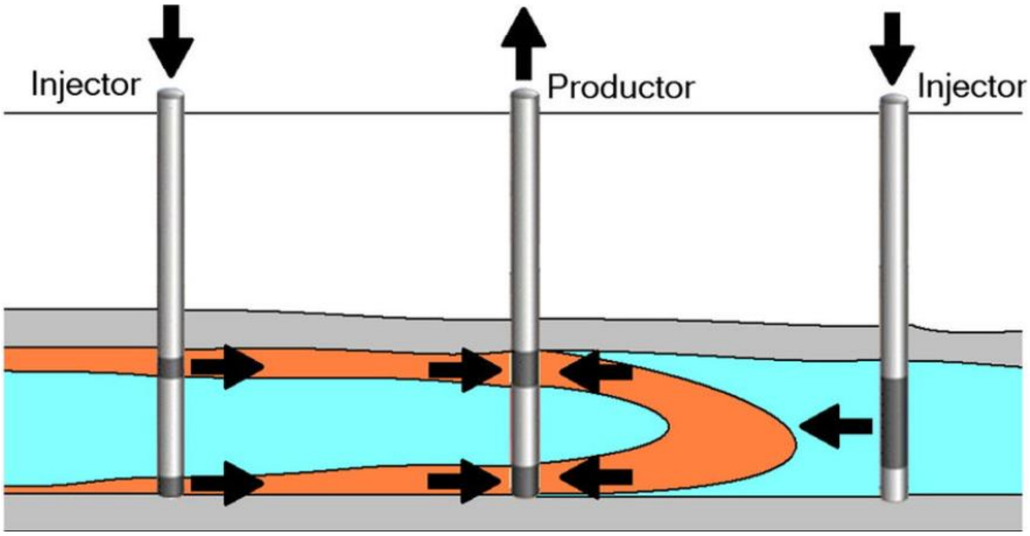

Fig. 1. Technical classification of wells for fluid injection and extraction

2 ANALYSIS OF METHODS

The article [1] examined the hydrodynamic process of ISL, which was used to extract valuable metals from ore deposits by treating the collector with acid. A MM based on filtration-convective and diffusion processes has been developed to thoroughly study, monitor, and forecast the object. This article was described by following MM. In this case, the propagation of the pressure field $H(x,y,t)$ was determined from the equations of the elastic filtration regime:

$$\begin{aligned}&\frac{\partial}{\partial x}\left[k(x,y)h(x,y)\frac{\partial H(x,y,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[k(x,y)h(x,y)\frac{\partial H(x,y,t)}{\partial y}\right]=\\&=\beta m(x,y)h(x,y)\frac{\partial H(x,y,t)}{\partial t}+f_1(x,y,t);(x,y)\in G,\end{aligned}\tag{1}$$

with initial conditions

$$H(x,y,t)=H^0(x,y);\quad t=0,\tag{2}$$

and boundary conditions

$$\alpha\frac{\partial H(x,y,t)}{\partial n}+(1-\alpha)H(x,y,t)=\varphi(x,y,t).\tag{3}$$

The propagation of the reactant field was determined by solving the convective diffusion equation with the corresponding conditions:

$$\begin{gathered}\frac{\partial}{\partial x}\left[D_{xx}(x,y)\frac{\partial C_1(x,y,t)}{\partial x}+D_{xy}(x,y)\frac{\partial C_1(x,y,t)}{\partial y}\right]+\\ +\frac{\partial}{\partial y}\left[D_{yy}(x,y)\frac{\partial C_1(x,y,t)}{\partial y}+D_{yx}(x,y)\frac{\partial C_1(x,y,t)}{\partial x}\right]-\\ -\frac{\partial(V_x(x,y,t)C_1(x,y,t))}{\partial x}-\frac{\partial(V_y(x,y,t)C_1(x,y,t))}{\partial y}=m_g\frac{\partial C_1(x,y,t)}{\partial t};\ (x,y)\in G,\end{gathered} \tag{4}$$

with initial and boundary conditions,

$$C_1(x,y,t)=C_1^0(x,y);\ t=0,\quad C_1(x,y,t)=0,\quad (x,y)\in G_k; \tag{5}$$

and also, internal conditions in wells:

$$\begin{aligned} C_1(x,y,t)&=C_{3i}; &\quad (x,y)\in G_0;\\ \frac{\partial C_1(x,y,t)}{\partial n}&=0; &\quad (x,y)\in G_z.\end{aligned} \tag{6}$$

The desired distribution of the concentration function of the useful component was determined by solving the following equation with the corresponding conditions:

$$\begin{gathered}\frac{\partial}{\partial x}\left[D_{xx}(x,y)\frac{\partial C_2(x,y,t)}{\partial x}+D_{xy}(x,y)\frac{\partial C_2(x,y,t)}{\partial y}\right]+\\ +\frac{\partial}{\partial y}\left[D_{yy}(x,y)\frac{\partial C_2(x,y,t)}{\partial y}+D_{yx}(x,y)\frac{\partial C_2(x,y,t)}{\partial x}\right]-\frac{\partial(V_x(x,y,t)C_2(x,y,t))}{\partial x}-\\ -\frac{\partial(V_y(x,y,t)C_2(x,y,t))}{\partial y}=m_c\frac{\partial C_2(x,y,t)}{\partial t}+\frac{\partial N}{\partial t};\ (x,y)\in G,\end{gathered} \tag{7}$$

with initial conditions

$$C_2(x,y,t)=C_2^0(x,y);\quad t=0, \tag{8}$$

and boundary conditions

$$\alpha\frac{\partial C_2(x,y,t)}{\partial n}+(1-\alpha)C_2(x,y,t)=\psi(x,y,t);\ \ (x,y)\in G_k, \tag{9}$$

and also, internal conditions in wells:

$$\begin{aligned} C_2(x,y,t)&=C_{4i}; &\quad (x,y)\in G_0;\\ \frac{\partial C_2(x,y,t)}{\partial n}&=0; &\quad (x,y)\in G_z.\end{aligned} \tag{10}$$

The equation of mass exchanged kinetics, which determined the rate of transition of a substance from one phase to another, has the following form:

$$\begin{gathered}\frac{\partial N(x,y,t)}{\partial t}=\gamma(C_1)f(C_2,N,t);\\ N(x,y,t)=N^0(x,y);\ \ t=o,\ \ (x,y)\in G,\end{gathered} \tag{11}$$

where, $H(x,y,t)$ is the variable head at the current point with coordinates (x,y) at an arbitrary point in time t(m), $\varphi(x,y,t)$ pressure value at the boundary (m), $k(x,y)$ is the filtration coefficient (m/day), $h(x,y)$ is thickness of the ore-bearing horizon (m), t is time, γ is the specific weight of leached reagent (*kg/m*^{3}), C is the concentration of minerals, G is the area of ore-bearing horizon, G_k is the boundary of the area, $M(x,z)=m\beta$, m is the porosity, β is the coefficient of elastic capacity (cm^3/kg), $f(x,y,t)=\sum_i^n q_i(t)\delta(x-x_i,y-y_i)$, $q_i(t)$ is well flow rates, δ is the delta function.

This model took into account key hydrodynamic indicators such as the filtration coefficient and changes in the porosity of the medium, as well as the process's dependence on pressure and kinetics. Protecting groundwater from sources of pollution was also a primary objective of the research. The problem was described by a multidimensional system of quasi-linear partial differential equations, making an analytical solution a challenging task. To address this issue, conservative difference schemes and basic flow-driving methods were applied. Subsequently, the two-dimensional problem was transformed into a chain of one-dimensional problems, and computational experiments are conducted on a cluster, with the results presented in graphical form. As the authors of the article claim, over time, this area expand depending on the permeability and porosity of the ore deposit and eventually reach the boundary of the ore layer after a certain period. It has been shown that for high-pressure values in the porous layer, the filtration coefficient and porosity should be calculated as an exponential function of pressure, while for low-pressure values, they should be calculated as a linear function of pressure.

In the [2] scientific research, expanding the application boundaries of the ISL method and exploring ways and tools for its used in deposits containing valuable minerals remain key tasks in this field. Monitoring groundwater levels in ISL regions is an important aspect of hydrogeological research. This monitoring allows for the assessment of filtration methods and rates of technological solutions, potential losses of technological solutions, hydraulic connections between productive aquifers and barren horizons, and the stability of the hydrodynamic regime in the studied area. The article's MM closely resembles the initial model, although some discrepancies exist. Instead of variable y and area G_k, z and Γ were used, (1) main equation and (2) initial condition were similar to equation of [1] work, and boundry conditions

$$H(x,z,t)=H_1(x,z,t), \quad (x,z)\in\Gamma. \tag{12}$$

The obtained difference problem was solved by the methods of longitudinal-transverse scheme and the flow variant of the sweep method.

Studies by Kazakh scientists [3-7] have demonstrate the relevance of investigating ore extraction process models using the ISL method, enabling the evaluation of the dynamics of this technological process. For instance, in [3], increasing the efficiency of the extraction process and solving associated challenges leds to growing attention on modeling the ISL process. In the article [3], a MM of the underground leaching technological system was constructed as follows:

$$W\frac{dH_{fw}}{dt}=Q_{fw}-Q_{ob}. \tag{13}$$

In equation (13) H_{fw} is leaching solution level in a filling well (m), W is capacity of the filling well (m^3), Q_{fw} is flow rate of the leaching solutions falling into the filling well (m^3/h), Q_{ob} is the leaching solutions flow rate from the filling well into the ore body through the well filter (m^3/h). If the filling well's flow rate equals the ore body's flow rate $\left(Q_{fw}=Q_{ob}\right)$, then the filling well's level remains stable $\left(H_{fw}=\ constant\right)$. The pressure generated by the filling well during steady-state operation is calculated as follows:

$$P_{fw}=H_{fw}\rho g. \tag{14}$$

The model of the depth pump is based on the pressure characteristic it generates, and takes the following form:

$$\Delta P_{pw}=a_0+a_1Q_{pw}+a_2Q^2{}_{pw}+a_3Q^3{}_{pw}. \tag{15}$$

In equations (14)-(15) ΔP_{pw} is pressure created by the pump (atm), Q_{pw} is flow rate (m^3/h), a_0,a_1,a_2,a_3 are coefficients of approximating. As a control criterion, the index reflecting the gradient of the pressure in the ore body from the pumping to the pumping well was selected, we write down the expression for the technological chain: a quench well, an ore body, a pumping well. Calculation of the pressure drop on the ore body was carried out according to the following scheme:

$$\Delta P_{ob} = R_{ob} \cdot Q_{ob}; \tag{16}$$

$$\Delta P_{ob} = P_{fw} - P_{ypw}; \tag{17}$$

$$P_{ypw} = (H_{ob} + H_0)\rho g; \tag{18}$$

$$H_0 = \left| P_{msd} - P_{lc} \right| \frac{1}{\rho g}; \tag{19}$$

in (16)-(19) ΔP_{ypw} is pressure on the sump of the depth pump (atm), H_0 is height of equilibrium point (m), P_{msd} is pressure at the mouth of the depth pump (atm), P_{lc} is pressure of the column of liquid in the pumping well (atm). The issues related to ISL for extraction, the current state and description of the technology, and its connection to management challenges are critically important. The article examines the presence of optimal pressure in pumping wells based on select criteria. It addresses finding the required flow rate at injection wells to ensure maximum productivity of the solution at the output. The study also involves determining the initial hydrodynamic parameters of wells and layers after specialized assessments, using deep well measurement instruments and information measuring tools for verification. The research focuses on simulating, predicting, and managing the ISL process for uranium by collecting relevant data. Using the Comsol Multiphysics software package, digital simulations of uranium ISL are conducted. The autors' previous studies evaluated key hydrodynamic characteristics of wells and layers, such as resistance coefficients and saturation recovery. This work, however, is connected with identifying changes in technological variables during operational processes in wells.

According to [4]-[5], with the ISL of metals, the question of the magnitude of the pressure at the pumping wells remains unresolved. So, with its increase, the production rate of wells increases, but the productivity of the solution at the outlet decreases. And with a decrease in pressure, the proportion of wells with which the concentration of metal in the productive solution increases decreases proportionately, but the rate of filtration of the solution decreases and, consequently, the rate of leaching of the metal. In this regard, there seems to be an optimal pressure on the pumping wells according to the selected criterion. The model provides a solution for such a required flow rate for a quench well providing the maximum output of the solution. In above research, the account of porosity level of the environment apparently led to the fact that the continuity equation for the continuous flow of homogeneous fluid:

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{W}) = 0, \tag{20}$$

will take the form

$$\frac{\partial (m\rho)}{\partial t} + div(\rho \vec{W}) = 0. \tag{21}$$

In (20)-(21) $\vec{W}$ is the filtration velocity vector. Regarding the Euler equations of motion, in the theory of filtration there are a number of assumptions that allow recoursing to Darcy's law:

$$\vec{W} = -\frac{k}{\mu} grad\,\overline{P}; \qquad \overline{P} = P + \rho g L. \tag{22}$$

In equation (22) ρ – density, P – pressure, g – acceleration of gravity, μ – dynamic viscosity, k – permeability, $\overline{P}$ - total pressure, L - well depth. The results of these articles, the technique for automated well placement has been developed, as well as the method for determining the optimal hexagonal well pattern. It has been emphasized that this technique can be applied to any reactive transport model of underground leaching, regardless of its complexity, taking into account gravity, clogging, and other chemical components.

The aim of studies [6] and [7] is to develop efficient well placement methods and manage operations for uranium extraction using the ISL method, as well as to determine the productivity of well networks. Production efficiency depends on the following factors: optimally selected operational regimes of the chosen well networks, including flow rates and well placement, as well as natural factors such as layer filtration properties, mineralization geometry, and concentration. In the ISL process, certain parts of the mineralization may perform inefficiently due to unique permeability and mineralization geometry, leading to oxidation with the leaching solution, formation of stagnant zones, or the spread of the solution beyond the operational block. For effective resource development, optimal well placement is determined based on balance zones (i.e., useful zones for extraction), results of hydrodynamic and reactive transport simulations, and economic assessments. Additionally, studies have explored graphics processing unit accelerated modeling of the uranium ISL process and Streamline-based reactive transport simulation. Over time, changes in flow rates within wells increase computational time, making simulation of the ISL process a resource-intensive challenge. The flow of leaching solution within porous media was governed by the mass conservation law and Darcy's law:

$$\frac{\partial(\varphi p_i)}{\partial t}+div(p_i\varphi\vec{u})=0; \tag{23}$$

$$\varphi\vec{u}=-\frac{K}{\mu}(grad(p)+p_i\vec{g}). \tag{24}$$

By applying the Darcy's Law in to the mass balance equation autors had been obtained the following hydraulic head equation:

$$div\left(\frac{K}{\mu}grad(p)\right)+\frac{gk}{\mu}\frac{\partial p_i}{\partial z}=-\frac{q}{p_i}, \tag{25}$$

where, $q=\sum q_{in}+q_{pr}=0$ with q is a quantity representing flow rates at well filters. Due to the fact that hexavalent uranium oxides dissolve better in sulfuric acid water solution, the reaction equation was written in following generalized form:

$$R+M\rightarrow P+W. \tag{26}$$

Reactive transport model of solution and dissolution of solid mineral was written in the following form:

$$\frac{\partial(\varphi p_i)}{\partial t}+\nabla*(\varphi c_i\vec{u})=\nabla*(\varphi D_i\nabla c_i)+W, \tag{27}$$

$$\frac{\partial((1-\varphi)c_i)}{\partial t}-W, \tag{28}$$

where, $w=-kC_RC_M$ characterizes change in mineral concentration as a result of reactant reacting with mineral. For detailed chemical kinetics equations (27) and (28) were expanded for each particular liquid and solid components.

In the extraction of ore from uranium deposits using the ISL method, the contributions of eastern countries scientists are significant, as demonstrated by several studies [8-12]. In [8], the hydrodynamics of groundwater in two adjacent well sites were simulated under various pumping-to-injection ratios. The goal is to determine the optimal ratio that prevents interaction between the groundwater flows of the two wells. Additionally, the sulfur isotope composition of groundwater in the two well sites is analyzed to verify the simulation results. The findings indicated that the flow rate at different points outside the edge of the drilling hole increased exponentially with the distance between the point and the edge. The flow line gradually extended beyond the drilling area as the leaching time increased. An optimal pumping-to-injection ratio of 1.003 was identified. After five years of leaching, the maximum distance between the moving front and the injection well reached 28.44 meters, demonstrating effective control of groundwater flow fields in the two wells. The process of ISL of uranium, the transport of groundwater in the ore-bearing aquifer is a confined three-dimensional unsteady flow, it could be described by the following differential equation:

$$\frac{\partial}{\partial x}\left[K_x\frac{\partial H}{\partial x}\right]+\frac{\partial}{\partial y}\left[K_y\frac{\partial H}{\partial y}\right]+\frac{\partial}{\partial z}\left[K_z\frac{\partial H}{\partial z}\right]+q_S=S_S\frac{\partial H}{\partial t}, \tag{29}$$

with the initial condition was given by (2) equation and boundary condition:

$$\begin{aligned} H(x,y,z,t)=H_1; \quad & (x,y,z)\in L_1, \\ H(x,y,z,t)=H_2; \quad & (x,y,z)\in L_2, \end{aligned} \tag{30}$$

$$\frac{\partial H}{\partial n}=0; \qquad (x,y,z)\in L_3. \tag{31}$$

In (29)-(31) $S_s\,[1/m]$ is the specific storage, $K[m/d]$ is the hydraulic conductivity, $H\ [m]$ is the water head at point (x,y,z) at time t, $q_s\,[1/d]$ is the source/sink term, which is the volume flows into or out of a unit volume aquifer, $H_0\,[m]$ is the initial water head, $H_1\,[m]$ is the initial groundwater head at the upstream well-site boundary L_1, $H_2\,[m]$ is the water head at the downstream well-site boundary L_2, *n* is the normal direction of the barrier boundary. Utilizing simulated groundwater level data, the groundwater flow velocity field at the well site could be determined using the following formula:

$$\overset{j}{\Delta} v = K\frac{I}{\overset{j}{\Delta} r};$$

$$\overset{j}{\Delta} t = \frac{\overset{j}{\Delta} r}{\overset{j}{\Delta} v} = \frac{\overset{j}{\Delta} r_2}{K*I}; \tag{32}$$

$$\overset{j}{T} = \overset{1}{\Delta} t^1 + \overset{2}{\Delta} t^2 + \cdots + \overset{j}{\Delta} t;$$

where, Δr_j is the distance between the centers of a calculation unit and its adjacent computing unit (m), Δv_j is the flow velocity of groundwater in two adjacent units (*m/d*), Δt_j is the time that water flows from one unit to its adjacent unit (*d*), *I* is the hydraulic gradient between two adjacent units, T_j is the time that water flows from a borehole to the *j* th unit (*d*).

In [9], the focus was on rationally controlling the leaching range, a critical indicator of the ISL process for uranium. However, optimal control technology is currently unavailable. To test and improve current control technologies in the industry, this study proposed an injection control regime for small flows around the well field. It developed a hydrodynamic model of the leaching range for eight different pumping and injection scenarios using groundwater modeling systems. The following mathematical model was considered in this research:

$$s(t) = s_1 + s_2 + \cdots + s_n, \tag{33}$$

$$s(t) = \frac{1}{4\pi T}\left[Q_1 W\left(\frac{r_1^2}{4a(t-t_1)}\right) + Q_2 W\left(\frac{r_1^2}{4a(t-t_2)}\right) + \cdots + Q_n W\left(\frac{r_n^2}{4a(t-t_n)}\right)\right], \tag{34}$$

when $\left(\frac{r_n^2}{4at}\right)$ approaches infinitely small, autors got:

$$s(t) = \frac{1}{4\pi T}\left[Q_1 \ln\frac{2.25a(t-t_1)}{r_1^2} + Q_2 \ln\frac{2.25a(t-t_2)}{r_2^2} + \cdots + Q_n \ln\frac{2.25a(t-t_n)}{r_n^2}\right], \tag{35}$$

where, Q_i is the flow rate of i-th well in the well-site, r_i is the actual distance from i-th drill well in the well-site to boundry point *p*, *T* is the specific moment for calculating the water level drawdown at the boundry point *p*, t_i is the specific moments when i-th drilling well in the well-site start to extract fluid, $i = \overline{1,n}.$

Study [10] described the ISL technology as a method for uranium mining where a chemical solution is injected through boreholes into ore-bearing layers. The solution moves along the ore layer under hydraulic gradient control and reacts with the ore, forming a uranium-containing solution. To minimize dead zones in the leaching process, each pumping and injection unit must achieve uniform leaching by the end of production. Furthermore, appropriate pumping and injection regimes should be established for each unit’s wells in the mining area. According to the percolation zone filled with liquid, mathematical modeling of this research:

$$\Delta q = -\left[\frac{\partial(\rho\upsilon_x)}{\partial x} + \frac{\partial(\rho\upsilon_y)}{\partial y} + \frac{\partial(\rho\upsilon_z)}{\partial z}\right]\Delta x\Delta y\Delta z\Delta t, \tag{36}$$

$$\Delta q = \frac{\partial}{\partial t}\left[\rho n \Delta x \Delta y \Delta z\right]\Delta t, \tag{37}$$

from (36) and (37) it follows that:

$$-\left[\frac{\partial(\rho\upsilon_x)}{\partial x} + \frac{\partial(\rho\upsilon_y)}{\partial y} + \frac{\partial(\rho\upsilon_z)}{\partial z}\right]\Delta x \Delta y \Delta z \Delta t = \frac{\partial}{\partial t}\left[\rho n \Delta x \Delta y \Delta z\right]\Delta t. \tag{38}$$

The seepage-dispersed solute transport equation describing the internal source-sink term:

$$\frac{\partial(nC)}{\partial t} = -\frac{\partial}{\partial x_i}(q_i C) + \frac{\partial}{\partial x_i}\left(nD_{ij}\frac{\partial C}{\partial x_j}\right) + q_S C_S. \tag{39}$$

In equation (39), the left side was the term of solute change with time, and the three terms on the right correspond to the solute seepage migration term, the dispersion migration term, and the source-sink term, respectively:

$$D = \alpha_L, \frac{\upsilon_L^2}{|\upsilon|} + \alpha_{TH}\frac{\upsilon_{TH}^2}{|\upsilon|} + \alpha_{TV}\frac{\upsilon_{TV}^2}{|\upsilon|} + D^*, \tag{40}$$

where, $\upsilon_x, \upsilon_y, \upsilon_z$ are percolation velocity components, ρ is density of liquid, Δq is the total mass difference between inflow and outflow within difference of time Δt, n is porosity, D is the diffusion coefficient, $\alpha_L, \alpha_{TH}, \alpha_{TV}$ are the dispersion of the solute in the vertical, horizontal, and vertical directions of the solution flow direction, respectively, and have the dimension of length, $\upsilon_L, \upsilon_{TH}, \upsilon_{TV}$ are the seepage velocity of water in the three respective directions, $|\upsilon|$ is the modulus of the seepage velocity vector; D^* is the effective molecular diffusion coefficient.

The research [11] in focused on the quantitative analysis of the impact of natural groundwater flowing into the flow field of ISL mines. A computational method has been developed to assess the effects of natural groundwater on the productivity of pumping wells in uranium ISL. The study establishes the “natural groundwater flow ratio” and corresponding formulas. Results showed that the variation in this ratio across different production stages of a mine or a single pumping well could be obtained using digital simulation of the mining area and neutral solution concentration values. Regulating the state, length, and injection regime of the filters in the leaching mine could control the amount of natural groundwater inflow, reducing fluid exchange between the leaching flow field and natural groundwater. The leaching range is crucial for leaching efficiency, production costs, and environmental impact. The MM of the work closely resembles the (29) equation, although some discrepancies exist. Instead of variable S, μ is used, initial condition is not differed from (2), and boundry conditionsis similar to (31). The solute transport equation is:

$$\frac{\partial}{\partial x}(q_x C) - \frac{\partial}{\partial y}(q_y C) - \frac{\partial}{\partial z}(q_z C) + q_S C_S = \theta\frac{\partial C}{\partial t}, \tag{41}$$

with initial condition was as (5), and boundry conditions was $C = 0$.

In [12], the dynamics of groundwater in a well site under various process parameters (e.g., drilling area and pumping-to-injection ratio) were simulated to determine the leaching range. The study also examined the control factors, evolution rules, and boundaries of leaching. Results has been revealed a specific water level depth outside the injection borehole, referred to as the "stagnation point," where the hydraulic gradient was zero. This indicate the furthest point to which the leaching solution could travel beyond the borehole. The connection line of all stagnation points quantitatively define the outer boundary of the leaching range from a hydrodynamic perspective. Reducing the spacing between wells increased the drawdown of groundwater and slightly expand the distance between the stagnation point and the edge injection well, marginally increasing the leaching range’s outer boundary. However, increasing the pumping-to-injection ratio significantly reduced the leaching range’s outer boundary. The MM of the work, initial condition and boundary condition exactly the same with [8]. These articles propose a method for quantitatively assessing the impact of natural groundwater on the production efficiency of pumping wells used for in situ uranium leaching. It has been shown that the concentration of the neutral solution extracted by each pumping well at different time intervals can be determined through numerical modeling of mining areas. Subsequently, based on the modeling results, the variations in the "flow ratio of groundwater" in mining areas or individual pumping wells at different production stages have been identified.

[13] article conducted on complex, highly interconnected technical systems (e.g., series-well – pump stations – reagent concentrations, etc.) that encompass multiple subsystems within technological washing processes. It has been demonstrated that all these subsystems are interconnected, and a disruption in the technological regime of at least one subsystem could led to the complete shutdown of the entire operational cycle. Therefore, significant attention was currently being paid to progressive methods for developing multi-component systems, one of which is the ISL method. Compared to other methods, the method is the most cost-effective and environmentally friendly approach, as its application does not lead to environmental degradation. This method is also widely used in the uranium mining industry, which holds high economic importance. The demand for energy derived from uranium, its primary source, is steadily increasing. This consideration highlights the importance of scientific research into effective methods for extracting valuable metals, particularly the application of the self-leaching process. The dissolution of useful components in the ground and the subsequent movement of the resulting compounds occur primarily in accordance with the laws of hydrodynamics, mass transfer, and chemical kinetics. The complexity of processes occurring under actual underground conditions necessitates the development of mathematical models and software to study the entire cycle of underground processes under real conditions and to make decisions aligned with objective management considerations. The primary goal of creating these models is to describe and predict specific objects and technological processes. Models and specific algorithms based on the mathematical interpretation of the problem assist in finding the necessary information for decision-making. Thus, developing models to address problems related to analysis and decision-making in managing technological ISL processes for ore extraction, as well as creating the corresponding computational algorithms and software, is highly relevant today. Its MM looks like (1) first-order partial differential equation with initial and boundary conditions are correspondingly (2) and (3). Following the resolution of problem (1) and the determination of head H, the filtration rate is calculated using Darcy's law (24). The reservoir's useful component concentration is determined using the convective diffusion equation. In this purpose (4)-(11) equations were used. The convective diffusion equation is considered to determine the concentration of a useful component in the reservoir:

$$\frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y}\right)-\frac{\partial(v_x C)}{\partial x}-\frac{\partial(v_y C)}{\partial y}-\gamma(C-C_m)=m\frac{\partial C}{\partial t}, \tag{42}$$

$$\frac{\partial N}{\partial t}\gamma(C)f(C,N,L,\Gamma), \qquad N(x,y,0)=N_0(x,y), \tag{43}$$

with initial, boundary and internal conditions:

$$C\left(x,y,0\right)=C_0;$$

$$\left(\alpha\frac{\partial C}{\partial n}+(1-\alpha)C\right)\Big|_{\Gamma}=\psi(x,y,t), \qquad C(x,y,t)\big|_{(x,y)=(x_i,y_i)}=C; \tag{44}$$

$$\frac{\partial C}{\partial n}\Big|_{(x,y)=(x_j,y_j)}=0.$$

In [14], the theoretical aspects of hydrodynamic processes in ISL for selective uranium extraction have been examined. The influence of the initial filtration gradient on the flow process of underground water in poorly permeable and poorly saturated uranium ores during selective uranium extraction into the solution has been studied. The hydrodynamic parameters of ISL for uranium ores with low water saturation have been analyzed. Equations and schematics for the consumption of injection and production wells used in the selective uranium leaching process have been developed. Due to the law of straight-line filtration, the consumption of water flowing into perfect wells is equal to:

$$Q=FK_{\Phi}J, \tag{45}$$

where, $F=2\pi xy,\ J=\dfrac{dy}{dx}.$

After substituting, the following differential equation was obtained:

$$Q=2\pi xy\cdot K_{\Phi}\cdot\frac{dy}{dx}. \tag{46}$$

The influence of the initial filtration gradient on the flow process of underground water flow of poorly permeable and poorly saturated uranium ores by the method of selective transfer of uranium to solution has been studied and the equation is the equation for calculating the full flow rate of a perfect well.

If we come next [15] work, reactive transport modeling is known to be computationally intensive when applied to 3D problems. An area of constant permeability with a uniform distribution of mineral-rock is considered and divided into hexagonal cells with injection wells (in red) located at its vertices and a pumping productive well at its center. Numerical modeling of the ISL requires finding solutions to a system of partial differential equations and generally consists of two major steps: (i) estimating the pressure *p* [*Pa*] and velocity u [*m/s*] fields using the Darcy's law and conservation laws; (ii) solving the equations governing mass transfer for the chemical components and the conservation of the solute mass:

$$u\phi = -\frac{k}{\mu}(\nabla p + \rho_l g), \tag{47}$$

$$\rho_l \nabla (u\phi) = q, \tag{48}$$

where, $\phi\left[m^3 \cdot m^{-3}\right]$ is porosity, *k* [m^2] is the intrinsic permeability, μ [Pas] is the dynamic viscosity, $\rho_l\left[kg/m^3\right]$ is the fluid density, $g\left[m/s^2\right]$ is the gravitational acceleration; ∇ [1/m] is the gradient operator, $\nabla\left[1/m\right]$ is the divergence operator, and $q\ \left[kg/m^3 \cdot s\right]$ is the flow rate sources. The fluid density and velocity are assumed to be spatially constant, so that ρl can be extracted from the divergence term and the time derivative of the velocity is zero $\left(\partial_t\left(u\phi\right)=0\right)$ in the conservation Eqation (48). Substituting Equation (48) into (47) gives the following stationary pressure flow mass conservative equation:

$$\nabla(u\phi) + \nabla\left(\frac{k}{\mu}(\nabla p + \rho_l g)\right) = q^*, \tag{49}$$

with $q^* = q/\rho_l\left[1/s\right]$ is the reduced source term which is positive for injection or negative for pumping. In the context of uranium mining using ISL methods, streamline-based simulations are utilized to calculate the total UO_3 recovery after leaching, required amount acid volume, and process duration. The simulations model the interactions between reagent and the uranium mineral, assuming simplified chemical kinetics of dissolution. The streamline-based approach simplifies the complex 3D numerical problem into a set of 1D numerical resolutions along the streamlines. By assuming constant average rock properties, ore grade, flow velocity, and porosity within the studied domain, this work has derived a one-dimensional analytical solution for the chemical concentration distribution along the streamlines.

In [16], a computational algorithm has been presented for simulating the evolution of the underground heap leaching process for gold ore. The model incorporates physicochemical data, geometric and operational parameters, such as the composition of the ore being dissolved, flow rates, and mixture concentrations, passivation parameters, ore volume distribution, average residence time of the solution in the heap, height, irrigated area, and weight of the ore in the heap. In this algorithm, the heap is divided into horizontal and flat layers of constant area. Gold recovery, residual cyanide concentration, and the enrichment of the saturated solution are calculated based on the interactions between these layers. A simplified model describing the solid-liquid reaction under diffusive control is employed to compute these variables. For each layer of the heap and each size class of the ore, the equation model is analytically solved at each time step. The flow in the heap is assumed to be unidirectional at a constant volumetric velocity, with concentrations of components in the flow varying over time. Axial and radial dispersions in the flow are neglected. The simulation demonstrates that the number of layers has minimal impact on the results, indicating the algorithm's stability and robustness. The average residence time of the solution in the heap and the apparent diffusion of cyanide through ore particles significantly affect the temporal evolution of gold recovery and its concentration in the saturated solution. These parameters are crucial for calibrating the model. When applied to an industrial case, the algorithm's results showed that the model is adequate for approximately predicting process efficiency. The MM of this problem:

$$\sigma_{HB}\varepsilon_{HB} = \frac{\tau Q}{H_{HB} S_{HB}}, \tag{50}$$

where, ε_{HB} is the heap porosity, σ_{HB} is the heap saturation, the average residence time of the solution in the bed of heap, Q is the rate of irrigation in the haep, $\overline{H}_{HB}$ is average heap height and $\overline{S}_{HB}$ is the average heap area.

There are a huge number of chemical and physical processes associated with the interaction of liquid and solid components. These processes include the ISL process of uranium, nickel, copper, precious metals and other solid compounds studied. In this [17] work, the main idea is the presence of new conditions on the free (unknown) boundary between the liquid and solid phases (“pore space - solid skeleton”). These conditions express the usual laws of conservation of mass and the derivation of a MM describing processes at the macroscopic level. The proposed method allows us to study how the dynamics of the free boundary depend on the propagation velocity of an inhomogeneous solution and external parameters (temperature, pressure and concentration of reagents). The MM of research: let us present a microscopic model of the leaching process. Let in dimensionless variables:

$$x \to \frac{x}{L}; \qquad t \to \frac{t}{T}; \qquad \upsilon \to \frac{T}{L}\upsilon; \qquad p \to Lg\rho^{0}p. \tag{51}$$

The behavior of the fluid in the pore space $\Omega_f(t)$ was described by the Stokes system of equations:

$$\alpha_{\mu}\Delta\upsilon - \nabla p = 0, \tag{52}$$

where, p is pressure and υ is speed of liquid. Let's use the continuity equation, where $\boldsymbol{v}\equiv 0$ and the liquid in the pores:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\chi\upsilon) = 0, \tag{53}$$

where, $\chi(x,t)$ is characteristic function of pore space.

The [18] research presents a MM describing the main hydrodynamic and physicochemical processes that determined the change in the state of the ore-bearing horizon and the behavior of the components of process solutions during sulfuric acid ISL of uranium. Hydrodynamic processes included changes in the pressure of formation waters, convective mass transfer, and hydrodynamic dispersion. Physicochemical processes include complexation, homogeneous and heterogeneous acid-base and oxidation-reduction processes, sorption-desorption, and dissolution-precipitation of minerals. The ISL method is used to develop exogenous uranium deposits in which the ore body is located in a highly permeable underground aquifer (productive horizon).

Uranium was extracted from the ore body using a system of (pumping and injection) wells combined into process units [19]. A leaching solution containing reagents capable of dissolving uranium minerals enters the productive horizon through injection wells. As a result of the physicochemical interaction of uranium minerals and host rocks with the leaching reagent, a uranium-containing productive solution is formed in the underground aquifer, which is extracted to the surface using pumping wells. The development of deposits using the ISL method has a lesser negative impact on the environment compared to traditional mining methods. There are no subsidences and disturbances of the earth’s surface, dumps of off-balance ores and waste rocks, or tailings storage facilities. However, when developing a deposit by the ISL method, the productive horizon is contaminated with leaching reagents and the products of their interaction with the host rock as a result of injection of process solutions [20]. To ensure the environmental safety of uranium mining by the ISL method, it is necessary to predict the change in the state of the productive horizon and the spread of pollutants during the development of the deposit and after its completion. A quantitative forecast can be obtained using MM [21]. However, most studies on the MM of uranium ISL solve problems of optimizing the operation of production units [22–26]. There are few studies devoted to modeling the migration of pollutants in the productive horizon after the completion of the development of a uranium deposit by the ISL method, and they are based on a simplified approach to the description of physicochemical processes [19, 27], which reduces the accuracy of the forecast. In this regard, the development of a MM that adequately describes the change in the state of the productive horizon and the behavior of pollutants during the process of uranium mining by the ISL method and after its completion is relevant.

In [28], ISL was a mining technique that extracts ore deposits without surface excavation. This process selectively transfers natural uranium ions into a productive solution within the subsurface. Wells were drilled into the ore body, leaching solution was injected, and the resulting metal-laden solution was pumped to the surface for processing. A schematic representation of the governing reaction describing uranium leaching via sulfuric acid solution was presented in:

$$UO_3 + H_2SO_4 = UO_2SO_4 + H_2O. \tag{54}$$

The useful element exchanged from solid form to liquid phase in reaction:

$$v_m M + v_r R = v_p P + v_w w, \tag{55}$$

where, M is gram-molecula of mineral (uranium) in solid phase, R is gram-molecula of reagent (sulfatic acid), P is gram-molecula of useful element of dissolved uranium, W is gram-molecula of by-product in liquid phase, v_r, v_m, v_p, v_w are stoichiometric coefficient of reagent, mineral, useful element and water, correspondingly. The system of equation was considered with following conditions: the medium was homogeneous and isotropic, the density of solution and layer were constant, and solution flow on the layer border didn’t exist. Then this filtration process was described by conservation law and Darcy law:

$$div\vec{V} + \sum_{i=1}^{nw} q_{si}\delta(\vec{x} + \vec{x_i}) = 0, \tag{56}$$

$$\vec{V} = -K \cdot gradH. \tag{57}$$

By substituting equation (56) into equation (57) and utilizing the relationship $h = p / \rho g$, autors derived the equation for hydrodynamic pressure *h* within the layer, expressed as following:

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) = -\sum_{i=0}^{nw} q_{si}\delta(\vec{x} - \vec{x_i}), \tag{58}$$

where, ρ isliquid density, K_{xx}, K_{yy}, K_{zz} are filtratsion coefficient (permeability) in the direction of *x,y,z* respectively:

$$\frac{\partial \overline{C_m}}{\partial t} = -\gamma\varepsilon C_R^0 \overline{C_m}\,\overline{C_r}, \tag{59}$$

$$\begin{gathered}\frac{\partial \varepsilon \overline{C_r}}{\partial t} = div(\varepsilon D grad \overline{C_r} - \overline{V}\,\overline{C_r} - v_1\gamma\varepsilon C_m^0 \overline{C_m}\,\overline{C_r} - \\ -\sum_d q\delta(x_d, y_d, z)\overline{C_r} + \sum_p q\delta(x - x_0, y - y_0, z),\end{gathered} \tag{60}$$

$$\frac{\partial \varepsilon \overline{C_p}}{\partial t} = div\left(\varepsilon D grad \overline{C_p} - \overline{V}\,\overline{C_p}\right) + v_1\gamma\varepsilon C_m^0 \overline{C_m}\,\overline{C_r} - \sum_p q\delta(x - x_0, y - y_0, z)\overline{C_p}, \tag{61}$$

where:

$$\overline{C_m} = \frac{C_m}{C_m^0}; \qquad \overline{C_r} = \frac{C_r}{C_r^0}; \qquad \overline{C_p} = \frac{C_p}{C_p^0}; \qquad v_1 = \frac{v_r R}{v_m M}; \qquad v_2 = \frac{v_p P}{v_m M}.$$

C_m is concentration of uranium, C_m^0 is initial content of mineral in layer; C_r is concentration of sulfatic acid in mixitura, C_r^0 is concentration of reagent on producing well, C_p is concentration of useful element (uranium) in mixture, $\overline{V}$ is filtration rate; *q* is debit of well, ε is porosity of layer, γ is coefficient, characterizing reaction rate, $D_{i,j,k}$ is hydrodynamic dispersion coefficient, defined as:

$$\begin{gathered}D_{xx} = \frac{\alpha_l u^2}{|V|} + \frac{\alpha_t v^2}{|V|} + \frac{\alpha_t w^2}{|V|} + D^*; \;\; D_{yy} = \frac{\alpha_l v^2}{|V|} + \frac{\alpha_t u^2}{|V|} + \frac{\alpha_t w^2}{|V|} + D^*; \\ D_{zz} = \frac{\alpha_l w^2}{|V|} + \frac{\alpha_t v^2}{|V|} + \frac{\alpha_t v^2}{|V|} + D^*; \quad D_{xy} = (\alpha_l - \alpha_t)\frac{uv}{|V|} + D^*;\end{gathered} \tag{62}$$

where, α_l is longitudinal dispersive and α_t is transverse dispersive.

Equations (59)-(61) were solved by initial and boundary conditions:

$$\begin{gathered} C_m\big|_{t=0}=C_m^0; \qquad C_r\big|_{t=0}=C_r^0; \qquad C_p\big|_{t=0}=C_p^0; \\ C_m\big|_S=0; \qquad C_r\big|_S=0; \qquad C_p\big|_S=0. \end{gathered} \tag{63}$$

The differential equation for hydraulic head (58) was solved by over relaxation iterative method. The filtration rate was defined from Darcy law using computed solution of head pressure. And transfer equation of reagent concentration in liquid phase (60), equation of useful element concentration in solid phase (59), and its transition to liquid phase (61) were solved by “Classics” scheme with upstream difference.

3 CONCLUSION

To conduct a comprehensive study, monitoring and forecasting of the state of ore deposits during the in-situ leaching process, it is necessary to develop new and improve the existing nonlinear mathematical models, efficient numerical methods and algorithms for solving a class of mass transfer problems in complex systems.

Based on a detailed analysis of literary sources, the task of further research is, development of geoinformation and hydrogeological models and a database necessary for the development of nonlinear mathematical models of the in-situ leaching process for the development of ore deposits, creation of a constructive system methodology for mathematical modeling of nonlinear complex dynamic problems of mass and heat transfer in ore deposits, development of nonlinear multidimensional mathematical models of in-situ leaching processes in complex multilayer porous media taking into account the hydrochemical properties of ore deposits, improvement of mathematical models of the in-situ leaching process in complex multilayer ore media taking into account the hydrochemical properties of the deposit, maintaining the ecological balance during mining, development of numerical algorithms and software environments for solving problems of the underground leaching process in complex multi-layered ore deposits, development of an effective numerical algorithm for solving the problem of the underground leaching process using the physical splitting method.

REFERENCES

[1] *Равшанов Н., Холматова И.И., Курбонов Н.М., Исламов Ю.Н.* Математическое моделирование процесса подземного выщелачивания с учетом изменения гидродинамических параметров пористой среды //Проблемы вычислительной и прикладной математики № 2(56) 2024.

[2] *Alimov I., Pirnazarova T., and Kholmatova I.* On a numerical method for solving the hydrodynamic problem of ISL // Journal of Physics: Conf. Series 1260 (2019) 102001

[3] *Mukhanov B., Omirbekova Zh., Orakbayev Y., Alimanova M., Pobrebnyak V.* Numerical modeling of ORE body and development hydrodynamic models of ISL process // 14th International Conference on Electronics Computer and Computation, ICECCO 2018, 2018, 8634795

[4] *Mukhanov B.K., Omirbekova Zh.Zh., Orakbayev Y.Zh., Kangtarbay M.B.*Modeling process of ISL of various regimes // The 15th international scientific conference information technologies and management 2017.

[5] *Mukhanov B.K., Omirbekova Zh.Zh., Usenov A.K., Wójcik W.* Simulating ISL Process Using COMSOL Multiphysics // INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2014, VOL. 60, NO. 3, PP. 213–217

[6] *Shayakhmetov N.M., Aizhulov D.Y., Alibayeva K.A., Serovajsky S., Panfilov I.* Application of hydrochemical simulation model to determination of optimal well pattern for mineral production with ISL // 9th International Young Scientist Conference on Computational Science (YSC 2020)

[7] *Tungatarovaa M.S., Kurmanseiita M.B., Shayakhmetov N.M.* GPU Accelerated Modeling of ISL Process and Streamline Based Reactive Transport Simulation // 9th International Young Scientist Conference on Computational Science (YSC 2020)

[8] *Yongmei Li, Chong Zhang, Zhipeng Tang, Chunguang Li, Zhenzhong Liu, Kaixuan Tan, Longcheng Liu.*. Hydrodynamics control for the well field of ISL of uranium // Nuclear Engineering and Technology 56 (2024) 4176–4183

[9] *Chong Zhang, Tingting Xie, Kaixuan Tan, Yixuan Yao, Yaan Wang, Chunguang Li, Yongmei Li, Ying Zhang, Hui Wang.* Hydrodynamic Simulation of the Influence of Injection Flowrate Regulation on ISL Range // Minerals 2022, 12, 787. https://doi.org/10.3390/min12070787

[10] *Chong Zhang, Kaixuan Tan, Tingting Xie, Yahui Tan, Lingdi Fu, Nan Gan, Lingzhen Kong.* Flow Microbalance Simulation of Pumping and Injection Unit in In Situ Leaching Uranium Mining Area // Processes 2021, 9, 1288. https://doi.org/10.3390/pr9081288

[11] *Qinci Li, Zhaokun Li, Tingting Xie, Ye Ding, Nan Gan, Yahui Tan, Chong Zhang.* Analysis of Natural Groundwater Flowing into the Flow Field of In Situ Leaching Mining // Processes 2023, 11, 471. https://doi.org/10.3390/pr11020471

[12] *Chong Zhang, Yongmei Li, Yuqing Niu, Kaixuan Tan, Tingting Xie, Yixuan Yao, Chunguang Li, Zhenzhong Liu.* Quantitative determination of the leaching range of ISL mining area by stagnation point // 1738-5733/© 2024 Korean Nuclear Society, Published by Elsevier Korea LLC.

[13] *Ismanova K.D., Isomaddinov U.M., Dedakhanov A.O.* Decision-Making Models for ISL Process Control. Nat. Volatiles & Essent // Oils,2021;8(4): 16235-16243.

[14] *Rajabboy Rozikov.* Research of hydrodynamic processes in ISL of uranium // E3S Web of Conferences 525, 01008 (2024).

[15] *Kurmanseiit M.B., Tungatarova M.S., Royer J.J., Aizhulov D.Y., Shayakhmetov N.M., Kaltayev A.* Streamline-based reactive transport modeling of uranium mining during ISL: Advantages and drawbacks // Hydrometallurgy 220 (2023) 106107.

[16] *LUIZ R.P. DE ANDRADE LIMA.* (MM)ling of Gold Ore Heap Leaching // Conference Paper. January 2002

[17] *Galtsev O.V., Galtseva O.A.* (MM) Of ISL At The Macroscopic Level // Mathematical physics. (MM).

[18] *Теровская Т.С., Кеслер А.Г., Носков М.Д.* Математическая модель изменения состояния продуктивного горизонта при серно-кислотном скважинном выщелачивании урана // Известия высших учебных заведений физика.

[19] *Акимова И.Д., Бабкин А.С., Воронцова О.М. и др .* Геотехнология урана (российскийопыт) // – М.: КДУ, 2017. – 576 с.

[20] *Белецки й В.И., Богатков Л.К., Волков Н.И. и др.* Справочник по геотехнологии урана // –М.: Энергатомиздат, 1997. – 672 с.

[21] *Румынин В.Г.* Геомиграционные модели в гидрогеологии // – СПб.: Наука, 2011. – 1158 с.

[22] *Носков М.Д., Гуцул М.В., Истомин А.Д. и др.* // Горный информационно-аналитический бюллетень // – 2012. – № 7. – С. 361–366.

[23] *Бабкин А.С., Гуцул М.В., Кеслер А.Г. и др.* // Изв. вузов. Физика. –2015. –Т. 58. –№ 12/3. –С. 3-8.

[24] *Гусаров М.А., Носков М.Д.* // Изв. вузов. Физика. – 2017. – Т. 60. – № 11/2. – С. 11–14.

[25] *Шрайнер А.Э., Носков М.Д., Носкова С.Н.* // Изв. вузов. Физика. – 2018. – Т. 61. – № 12/2. – С. 140–144.

[26] *Разуваева Т.В., Мырзабек Г.А., Копбаева М.П.* Актуальные проблемы урановой промышленности: сб // трудов IX Междунар. науч.-практич. конф. – 2019. – С. 322–328.

[27] *Бураков М.М., Павличенко Л.М.* Актуальные проблемы урановой промышленности // трудов VI Междуна. науч.-практич. конф. – 2010. – С. 335–345.

[28] *Libayeva K.A., Kulzhabekov A.B., Kaltayev A.* The modeling of uranium extraction process by the in situ leaching method // WIERTNICTWO NAFTA GAZ. TOM 27. ZESZYT 1-2.2010

Received: December 18,2024

Citation: *Ravshanov N., Usmonov L.* 2025. Review of research on mathematical modeling of the process of in-situ leaching of mineral resources. *International Journal of Theoretical and Applied Issues of Digital Technologies*. Volume 8, Issue 1: pp 22-36. https://doi.org/10.62132/ijdt.v8i1.230.

ОБЗОР ИССЛЕДОВАНИЙ ПО МАТЕМАТИЧЕСКОМУ МОДЕЛИРОВАНИЮ ПРОЦЕССА ПОДЗЕМНОГО ВЫЩЕЛАЧИВАНИЯ ПОЛЕЗНЫХ ИСКОПАЕМЫХ

Равшанов Н.[1], + *Усмонов Л.*[1]

[1] Научно-исследовательский институт развития цифровых технологий и искусственного интеллекта, Ташкент, Узбекистан

+uslochinbek@gmail.com

Аннотация. В работе рассматривается резюме проделанной работы по применению методов подземной добычи при закладке рудных месторождений. Кроме того, приводится основная информация для будущих исследований. Анализ проведенных численных расчетов показал, что на начальном этапе разработки рудного месторождения активная реакция реагента, подаваемого с помощью нагнетательных скважин, происходит вокруг скважин, и как показывают численные расчеты в компьютерной системе, эта область расширяется с течением времени в зависимости от проницаемости и пористости рудного месторождения. В этом исследовании обширный обзор литературы и анализ пробелов используются для выявления ограничений и возможностей для дальнейшего исследования метода подземного выщелачивания (ПВ) для добычи руды по комплексному планированию разработки ресурсов. Более того, нашей целью является совершенствование методов добычи различных полезных ископаемых методом подземного выщелачивания и создание для него математической модели на основе выполненной работы. Считаем, что добыча полезных ископаемых, особенно руд, всегда была и будет актуальной экономической и политической проблемой. Поэтому спрос на исследования в этой области будет продолжать расти.

Ключевые слова: подземное выщелачивание, математическая модель, процесс добычи, моделирование, месторождение урана, закон Дарси.